Position Estimation in Ad hoc Wireless Sensor Networks with Low Complexity

Jan Blumenthal, Frank Reichenbach, Dirk Timmermann

Institute of Applied Microelectronics and Computer Science
University of Rostock
{jan.blumenthal;frank.reichenbach;dirk.timmermann}@uni-rostock.de

Abstract– In sensor networks, randomly distributed sensor nodes have to determine its own positions to assign measurements to locations. Due to limited resources of the nodes, resource-aware positioning algorithms are required.
In this paper, we present a solution to minimize the positioning error of the "Weighted Centroid Localization" - algorithm (WCL) using hop count determination. Compared to other approximative algorithms, WCL achieves a remarkable average positioning error below 8%. Further, we introduce abstract distances to support different types of distance information, e.g. hop count determination or distance measurements. Finally, we present an analytical equation to compute a qualified transmission range in scenarios with hop count determination.

1 Introduction

New technologies and advances in modern communication technologies lead to development of extreme small, cheap, and smart sensor nodes. These nodes consist of sensors, actuators, a low power processor, small memory, and a communication module. The nodes measure conditions of the environment, precalculate, aggregate, and transmit the data to a base station. Thousands of these nodes form a large wireless sensor network to monitor huge inaccessible terrains [1,2]. Due to the desired node's size of some millimeters, the dimensions of the communication module and the battery are critical. Consequently, the tightest resource within a network is the available energy. Therefore, it is essential to use low power optimized algorithms beside power saving hardware components.
Simple uncoordinated seeding of nodes yields a stochastic distribution of nodes after deployment phase. This impedes the assignment of a measured value to its location. Due to this fact, a position determination of all nodes is necessary which, however, consumes additional energy for calculations and data transmissions.
This paper presents optimizations of the "Weighted Centroid Localization" (WCL). It considers an abstract distance determination and the derivation of a recommendable transmission range of beacons.
This paper is subdivided as follows. In Section 2, we categorize localization algorithms. Then, we explain the "Weighted Centroid Localization" algorithm (WCL) in Section 3. Next in Section 4, we present a solution for an optimizing problem concerning overlapping transmission ranges in WCL followed by a simulation in Section 5. Finally, the paper ends with a conclusion in Section 6.

2 Localization in Sensor Networks

Localization algorithms are mainly classified into approximative and exact localization, respectively (Fig. 2). Exact localization is based on precise measurements of distances or angles between sensor nodes not knowing their own position and nodes with preinstalled localization systems. These methods facilitate high precision of position determination but results in extensive calculations and partly high network traffic [4]. However, exact positions are not always required. Often, a deviation of 8% is sufficient which can be achieved by using approximative algorithms [5,6,7,8]. These algorithms merge low calculation requirements with less network traffic. The presented algorithm WCL is a representative of the approximative algorithms even though it achieves nearly the precision of exact localization algorithms.

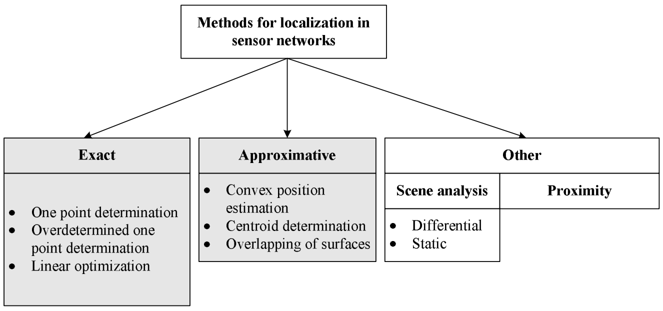
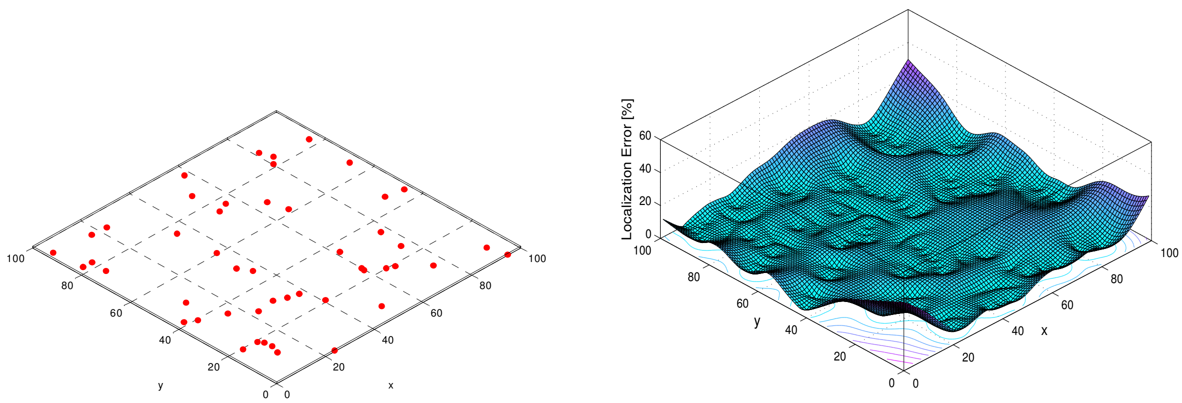


Fig. 2 Classification of methods for localization in sensor networks

3 Weighted Centroid Localization

3.1 Coarse Grained Localization

A sensor network with a total number of *k* nodes consists of *u* sensor nodes and *b* beacons (*b*<<*u*). Beacons are equipped with more efficient hardware and a localization system (e.g. GPS or Galileo [3]), whereby they are able to determine their own position. In practice, the positioning error of these localization systems depends on the quality of the used localization devices. Thus, the localization



a) **b)**

Fig. 1 a) Sensor network with 60 uniformly distributed beacons (solid red points), b) Localization error of the „Weighted Centroid Localization" algorithm (WCL) in a sensor network with *100x100* sensor nodes and the same beacon distribution from a)

error varies between meters and centimeters. Furthermore, determined positions are assumed to be exact. Sensor nodes consist of minimal hardware and do not know their own position. During deployment, sensor nodes and beacons are uniformly distributed over an area of interest (Fig. 1a).
Publicized algorithms such as "Coarse Grained Localization with Centroid Determination" CGLCD calculate centroids to estimate their own position [5]. In the first phase, all beacons send their position $B_j(x,y)$ to all sensor nodes within their transmission range. In the second phase, all sensor nodes calculate their own approximative position $P_i'(x,y)$ by a centroid determination from all n positions of the beacons in range (1).

$$P_i'(x,y) = \frac{1}{n}\sum_{j=1}^{n} B_j(x,y) \quad (1)$$

$P_i'(x,y)$ = Approximated position of sensor node i (CGLCD)
n = Number of beacons in range of sensor node i

The localization error $f_i(x,y)$ is the distance between the exact position $P_i(x,y)$ and the approximated position $P_i'(x,y)$ of sensor node i (2).

$$f_i(x,y) = \sqrt{(x'-x)^2 + (y'-y)^2} \quad (2)$$

3.2 Abstract Distances

The approach of the WCL algorithm is to consider beacons next to sensor node i more than remote beacons. In addition, the algorithm does not require very high precision of input values to converge. In WCL, the measurement unit to distinguish neighboring beacons from remote beacons is not statically defined. Therefore, different types of measurement units are applicable.
A typical criterion to distinguish neighboring beacons from remote beacons is the distance between a beacon and a sensor node. This distance originates either from the "Received Signal Strength Indicator" (RSSI) of a beacon's message or wave's time of flight. The RSSI decreases with a growing distance to the beacon. Alike in CGLCD, RSSI measurements require no additional effort in the hardware of beacons. Many publications claim that using RSSI is complicated due to unprecise measurements. Therefore, we gathered the signal strength with sensor node CC1010 manufactured by Chipcon. Fig. 3 visualizes the received signal strength in azimuth plane. The resulting graph is alike a circle. Thus, a distance determination by means of RSSI is possible.

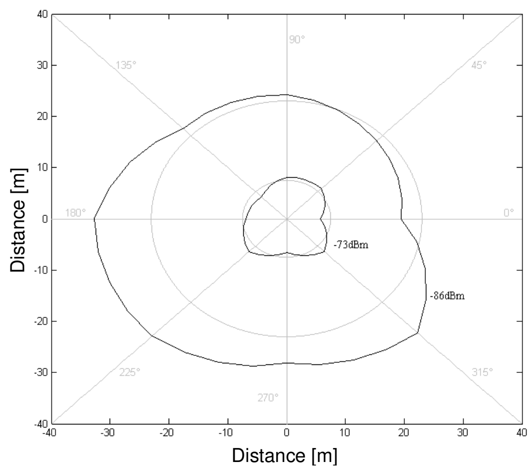


Fig. 3 Received signal strength (azimuth plane) of sensor node Chipcon CC1010EM (868MHz, outdoor)

Another criterion to consider the distance is the hop count between a beacon and a sensor node [13,14]. To determine the hop count, a flooding is initiated by a sensor node *i* to request beacons. Each beacon in range replys the request with hop count 1. Then, all beacons broadcast the original request to their neighbors. If beacons reply to this request, the beacons in range will forward this reply to sensor node *i* with hop count 2. This procedure continues until all beacons have replied. To determine a correct hop count, the transmission range of beacons must be adjusted so that each beacon preferably achieves its direct neighbors only as described in Section 4.
WCL is divided into three phases (Fig. 4). In the first phase, sensor nodes and beacons are distributed randomly. Next in the second phase, distances between beacons and sensor nodes are determined. Currently, two methods are evaluated – distance measurement and hop count determination. Both methods provide valid distance information between sensor node *i* and beacon *j*. In WCL, this information is defined as distance d_{ij}. During phase 2, it is not necessary to determine distances to every beacon. More precisely, it is sufficient that:

- beacons $b(i)$ used to determine distance d_{ij} are within a circle r_{max}
- beacons $b(i)$ are distributed uniformly within r_{max}
- distances d_{ij} must increase linear and must be independent of the angle around sensor node *i*.

3.3 WCL Algorithm

Starting point of WCL is the centroid determination (1). Due to interferences, obstacles, and hardware restrictions, the measured distances are very inaccurate. Hence, distances are used only as additional input for the localization algorithm [9,11,12]. Thus, distances must not impact the position determination very excessive. Therefore, WCL uses distance information as a weight w_{ij}. Small distances to neighboring beacons lead to a higher weight than to remote beacons. Further, every coordinate of a beacon's position obtains a weight depending on the distance $w_{ij}(d_{ij})$. The weight w_{ij} is inserted in (1) and forms the equation of the weighted centroid determination (3).

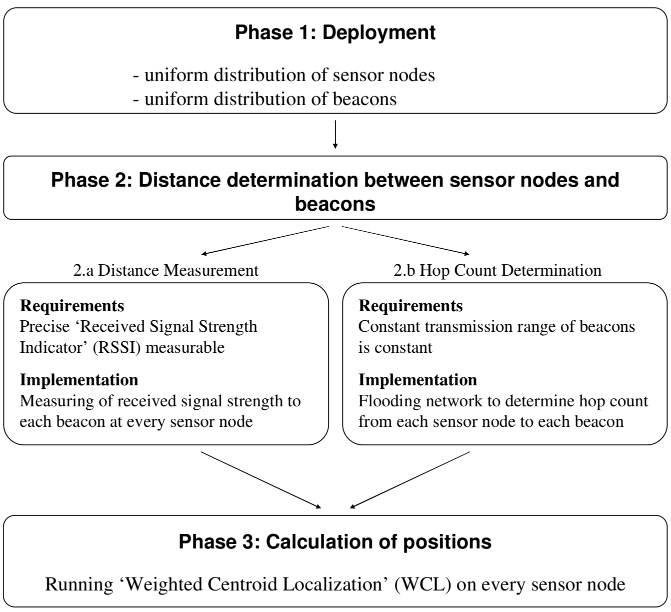


Fig. 4 Localization of sensor nodes using WCL with 2.a) distance measurements or 2.b) hop count determination

$$P_i''(x,y) = \frac{\sum_{j=1}^{n} \left(w_{ij} \cdot B_j(x,y) \right)}{\sum_{j=1}^{n} w_{ij}} \tag{3}$$

$P_i''(x,y)$ = approximated position of sensor node i (WCL)
w_{ij} = weight of distance d_{ij}
n = number of beacons in range

Fig. 5 shows the approximated position of sensor node P_i' determined by a simple centroid calculation (CGLCD) with positions of four beacons $B_1 ... B_4$. The weighted centroid determination of WCL considering distances $d_{i1}...d_{i4}$ results in the approximated position P_i''. Distance d_{i1} is higher weighted than d_{i4} and significantly more weighted than distances d_{i3} or d_{i2}. Consequently, the approximated P_i'' moves considerable to B_1 and slightly to B_4 as well as to the exact position $P_i(x,y)$ and thus $f_i(x,y)$ decreases.

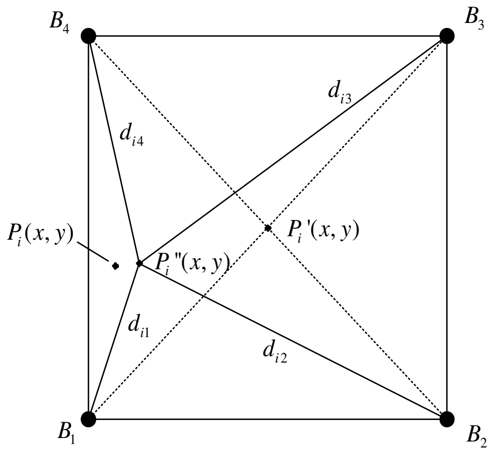


Fig. 5 Localization determination of sensor node $P_i(x,y)$ with CGLCD ($P_i'(x,y)$) and WCL ($P_i''(x,y)$) with measured distances d_{ij} in a grid-aligned network of beacons

4 Minimizing Positioning Error using Hop Count Determination

In WCL, the hop count is defined as number of hops between a sensor node and a beacon as visualized in Fig. 6a. To minimize the positioning error, the hop count resolution is required to be high. Thus, the transmission range of sensor nodes should be as small as possible. To determine this transmission range, we define a mathematical model and only focus on field A of the whole network.

For our next theoretical considerations, we assume a quadratic field with side length w. All sensor nodes, furthermore denoted as points with transmission ranges denoted as circles, are uniformly placed over the quadratic area $w \cdot w$. Every point is the center of a circle with a definite radius r, which corresponds to an ideal transmission range.

The uniform placement of the nodes forms a solid grid. Every point possesses the same distance a to each of its neighbors. Fig. 6b illustrates the described initial configuration. Every circle must have the same radius to support the hop count determination.

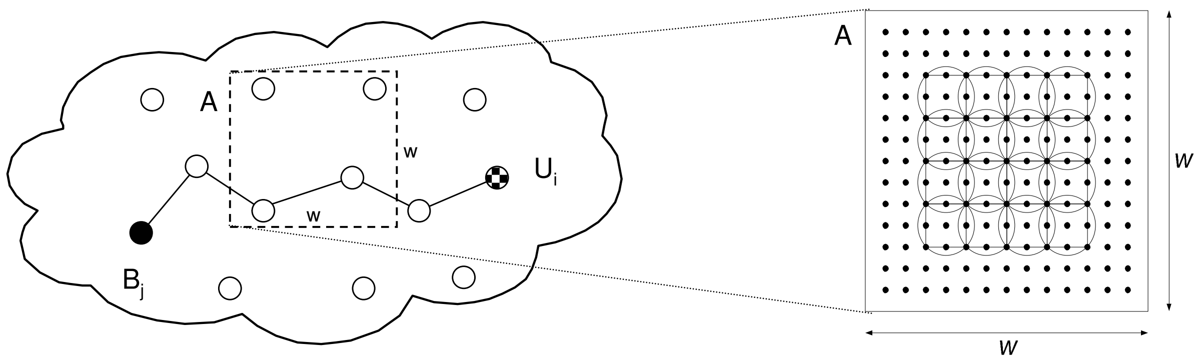


Fig. 6 a) Example determination of hop count between beacon B_j and sensor node U_i b) Theoretical model: quadratic field $w{\cdot}w$ with uniformly placed sensor nodes (points) and their transmission ranges (circles).

In a quadratic field with side length *w* and *n* points, the distance between all sensor nodes is defined as:

$$a = \frac{w}{\sqrt{n} - 1} \tag{4}$$

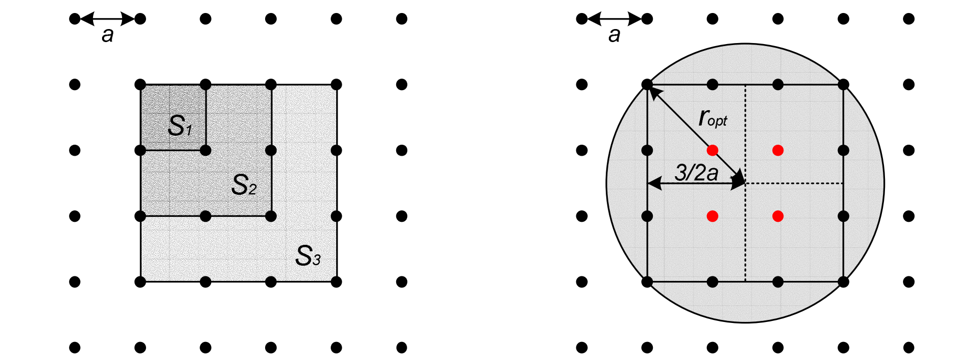
The radii of the circles have to satisfy the following conditions.

1. At the minimal radius, every point must have neighboring points in its circular area. Otherwise, sensor nodes within the network are unconnected. Therefore, we define the connectivity of the network. To form a network without isolated parts, the connectivity must be one hundred percent.
2. The second condition limits the radius of being too high. We demand a very small radius to reach a high resolution of the hop count. For example, if the radius is too high, only a few hops suffice to reach one border of the network from the opposite one. Thus, the hop count is very small and useless to approximate the distance. This results in an undesired high localization error.

Summarized, we need the smallest radius r_{opt} at the highest connectivity. The postulated conditions require the determination of an optimal radius. The problem of determining an optimal radius is replaced by the following simple approach. To reach a high connectivity, all circles must cover the whole quadratic field $w{\cdot}w$. This assumption results in a mathematical coverage problem which cannot be trivially solved with circles. Therefore, we substitute the circle with an adequate geometric surface – the square.

Arranging many small squares in a large square as displayed in Fig. 6b is definite. Hence, the area coverage of many small squares is completely without gaps or overlaps. To substitute the circle with a square clearly, the square must be put on top of the circle with all four corners tangenting the circle border (Fig. 7b). To reach a high connectivity, a square requires more than one neighboring point. Although usually four points are on the square's border, we do not consider these points. Therefore, we focus on the determination of the slightly increased side length *a* of the squares. Importantly to prevent a coverage with overlaps or gaps, the side length of the square must only be a multiple of *a*.

The first and trivial case is side length 1*a*. Fig. 7a demonstrates that in case S_1 no point is on the square. Hence, side length 1*a* is not qualified for further considerations. By stepwise increasing the side length, next case is 2*a*. The formed square S_2 contains exactly one point which conflicts with the basic condition of requiring more then one point. In case 3 with the side length 3*a*, four points are on the square, which is sufficient and meets all conditions.



a) b)

Fig. 7 a) All three cases with different side lengths of the squares b) Case 3 with the derivation of the optimal radius.

Side length 3a is the smallest radius sufficing the two introduced conditions. Therefore, we stop incrementing and analyse the current configuration.

Now, the square is resubstituted back to a circle. The optimal radius is calculated by the theorem of Pythagoras (6) as visualized in Fig. 7b.

$$\left(r_{opt}\right)^2 = \left(\frac{3}{2}a\right)^2 + \left(\frac{3}{2}a\right)^2 \tag{5}$$

***a* = distance between sensor nodes**

The equation to compute the smallest radius (6) which satisfies a connectivity of 100% is formed by simplifying and reforming (5). With (6), we derived a simple equation for determining the optimal radius analytically.

$$r_{opt} = \frac{\sqrt{18}}{2}a \tag{6}$$

5 Simulation

In this section, we demonstrate that Equation (6) to determine the optimal radius meets the two conditions introduced in Section 4. Therefore, we calculated the optimal radius at different numbers of sensor nodes. The radii are listed in Tab. 1. Moreover, we simulated the WCL algorithm with hop count determination and compared the results with calculated ones obtained by (6). The optimal radius in the simulation was computed at the minimum of the localization error, where all sensor nodes are connected to the sensor network. The comparison demonstrates a good correlation of the simulated and the calculated radii.

In addition, the connectivity is shown in Fig. 8a, where the *y*-axis is equal to (1-connectivity). Due to that fact, a high value assigns an unconnected network. At a very small radius, no sensor node reaches even one neighbor; the network is completely unconnected. With an increasing transmission range, the connectivity rises and at a specific range all nodes are connected to the network. The different numbers of sensor nodes only cause a shifting of the curves along the *x*-axis, because more nodes increase the hop count resolution. For a direct comparison with results obtained by (6), the vertical dotted lines mark the calculated optimal radii. All optimal radii guarantee a full connected network. Thus, the first condition demanding a high connectivity is met.

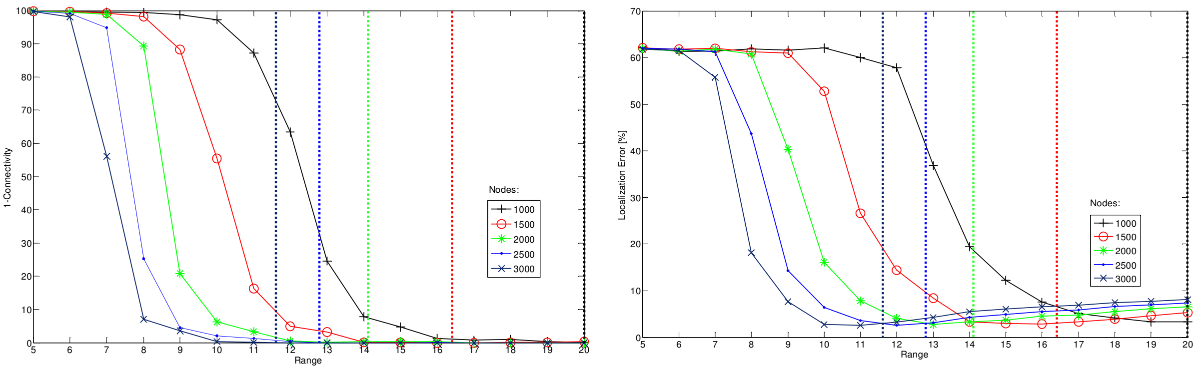
Sensor nodes u	Optimal radius r_{opt} (Equation (6))	Optimal radius r_{opt} (Simulation)
1000	20,1244	~20
1500	16,4315	~16
2000	14,2301	~14
2500	12,7278	~13
3000	11,6188	~12

Tab. 1 Optimal radius r_{opt} determined by the simulation or Equation (6)

Moreover, Fig. 8b shows the relative localization error depending on the field width. The error drops fast in the first interval, reaches its minimum and rises very slow and nearly linear. An increasing number of sensor nodes improves the accuracy and shifts the curves to smaller transmission ranges. This effect results from smaller distances between sensor nodes in a quadratic field A with a constant width w=100. Additionally, the dotted lines in Fig. 8b mark the calculated optimal radius. The optimal radius characterizes the points with a minimal localization error at a connectivity of 100%. Thus, condition 2 is also fulfilled and approves the correctness of Equation (6), within the simulation environments. Finally, the simulation results demonstrate the small localization error of WCL, even with the hop count determination.

6 Conclusion

In this paper, we presented a solution to minimize the positioning error of the "Weighted Centroid Localization" - algorithm (WCL) using hop count determination. WCL uses abstract distances as weight to approximate the position of sensor nodes with a minimal localization error of maximal 8%. The algorithm is flexible, due to the different substitutions of the weight either as RSSI or hop count. Moreover, we derived a simple analytical equation to calculate an optimal transmission range for different sensor network configurations with hop count determination. At this transmission range, all sensor nodes are connected and there are no isolated network parts anymore. The algorithm scales well in large sensor networks and requires minimal memory, processor time, and therefore energy.



a) b)

Fig. 8 a) Connectivity, and b) Localization error of the „Weighted Centroid Localization" algorithm (WCL) in a sensor network with side length w=100, different amounts of sensor nodes over an increasing transmission range and b=49 beacons; the vertical dotted lines correspond with the calculated qualified radii of Equation (6)

References

[1] Ian F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, “A survey on sensor networks”, *IEEE Communications Magazine*, pp. 102- 114, August 2002.

[2] H. Karl, A. Willig, “A short survey of wireless sensor networks”, *TKN Technical Report* TKN-03-018, Berlin, October 2003.

[3] J. Gibson, “The mobile communications handbook”, 2. Auflage, CRC Press, United States of America, 1996.

[4] A. Savvides, C. C. Han, and M. B. Strivastava, “Dynamic fine grained localization in ad-hoc networks of sensors”, *Proceedings of the 5th International Conference on Mobile Computing and Networking*, Mobicom 2001, pp. 166-179, Rome, Italy, July 2001.

[5] N. Bulusu, J. Heidemann, and D. Estrin, “GPS-less low cost outdoor localization for very small devices”, *IEEE Personal Communications Magazine*, 7(5):28–34, October 2000.

[6] L. Doherty, K. S. J. Pister, and L. El Ghaoui, “Convex position estimation in wireless sensor networks”, *Proceedings of IEEE Infocom*, 2001.

[7] H. Tian, “Range-free localization schemes for large-scale sensor networks”, MobiCom ’03, San Diego, California, USA, Copyright 2003 ACM 1-58113-753-2/03/0009, Department of Computer Science, School of Engineering, University Virginia, September 14-19, 2003.

[8] S. Capkun, M. Hamdi, and J.-P. Hubaux, “GPS-free positioning in mobile ad hoc networks”, *Cluster Computing*, 5(2): 157-167 (2002).

[9] F. Reichenbach, „Localization in wireless ad hoc networks“, *Master Thesis*, University of Rostock, Germany, 2004.

[10] J. Blumenthal, F. Reichenbach, M. Handy, and D. Timmermann, “Optimal adjustment of the coarse grained localization-algorithm for wireless sensor networks”, Invited Paper, *Proceedings of 1st Intl. Workshop on Positioning, Navigation, and Communication WPNC'2004*, Hanover, Germany, March 2004.

[11] S. Tschumi, “Positioning in mobile ad hoc networks”, *Semester Thesis*, ETH Zürich, Switzerland, 2002.

[12] S. Feldmann, “An indoor Bluetooth-based positioning system: concept, implementation and experimental evaluation”, *ICWN'03*, Las Vegas, USA, Institute of Communications Engineering, Hanover, June 23-26, 2003.

[13] G. Gramlich, “Numerical mathematics with matlab – An introduction for scientists and engineers“, dpunkt.verlag, 2000.

[14] C. Savarese, “Robust positioning algorithm for distributed ad-hoc wireless sensor networks”, Technical Report, Delft University of Technology, 2001