SBCL - Improved Centroid Estimation

Ralf Behnke, Jakob Salzmann, Dirk Timmermann
Institute of Applied Microelectronics and Computer Engineering
University of Rostock
18057 Rostock, Germany
{*ralf.behnke, jakob.salzmann, dirk.timmermann*}*@uni-rostock.de*

***Abstract*—Localization of sensor nodes is one of the key issues in Wireless Sensor Networks. It is a precondition for a variety of applications, as well as geographic clustering and routing. A simple approach for coarse grained localization is Centroid Localization (CL) which was firstly presented by Bulusu et al. and assumed regularly arranged beacons. Unfortunately, CL has a biased error whenever exactly three beacons are in range or the beacons are situated at the edges of the sensor field. In this work, we investigate the bias and its impacts on localization error. With our shape-based CL algorithms, we present possibilities to reduce this error and improve the localization accuracy of CL. However, since our new approaches introduce more complexity to CL, an analysis of the complexity of the algorithms is presented.**

***Keywords*-Wireless Sensor Networks, Centroid Localization, Optimization**

I. Introduction

Recent technological advances led to the development of tiny wireless devices, which are able to sense the environment, compute simple tasks and exchange data among each other. Interconnected assemblies of such devices, called Wireless Sensor Networks (WSNs), are commonly used to observe large inaccessible areas [1]. For the majority of WSN scenarios the collected data need to be combined with geographic information to make them useful. Moreover, localization in WSN is fundamental for geographic clustering [2] as well as geographic routing [3][4]. Due to existing limitations in terms of size and energy consumption, localization within the network is preferred over utilizing a commercial positioning system like GPS [5]. Consequently, a typical approach is to assume that only a small number of selected nodes know their exact position a priori or obtains it using common positioning systems. Then, all other nodes calculate their position with the help of these beacon nodes.

The accuracy of localization techniques ranges from high precision, commonly based on solving a set of nonlinear equations, to low precision. Bulusu et al. divided localization into coarse-grained and fine-grained algorithms and proposed a coarse grained localization algorithm, which needs only a minimum of computations, called Centroid Localization (CL) [6]. In CL, all non-localized nodes calculate their position as the centroid of all received beacon's positions. In [7] Blumenthal et al. studied the precision of CL. In this work, an optimal ratio between beacon distance and communication range for regularly arranged beacons has been found. Unfortunately, Blumenthal did not consider the inherent bias of CL.

In this paper, we present our investigations concerning the bias of CL and the achievable improvement when correcting the bias, using Shape-Based Centroid Localization (SBCL). Our approach will be evaluated in terms of accuracy and complexity. We furthermore extend our approach to localize nodes at the borders of the network with improved accuracy.

The remainder of the paper is organized as follows. In Section II, the original CL algorithm is explained. In Section III, we discuss the parameters that influence the precision of CL. In Section IV, we develop our approaches to reduce the bias, using SBCL and present results of the corresponding computer simulations. Section V compares the performance of our approach with CL. Finally, Section VI closes with a conclusion and future work.

II. Related Work

This Section briefly reviews the original CL [6], and existing improvements found by Blumenthal et al. [7]. Our approaches to improve centroid localization can be regarded as an enhancement of this algorithm. Therefore, this Section shortly reviews the CL algorithm as well as the analyses performed by Blumenthal et al.

We assume the presence of location-aware nodes, called beacons, which serve as anchor points for the localization of other location-unaware nodes which we refer to as unknowns. Algorithms that are mentioned in this paper will be explained and have been analyzed for a regular deployment of four beacon nodes forming the corners of a square.

A. Centroid Localization

The pure CL does not utilize the Received Signal Strength Indicator (RSSI) or any other parameter, indicating the distance between a beacon node and an unknown. The only kind of distance information used in CL is the binary information whether the unknown is in the communication range of a beacon or not. CL assumes a circular area with the center being the beacon's location as communication range,

9781-4244-3941-6/09/$25.00

i.e. unit disc graph model. Figure 1 depicts the communication ranges of four beacons, arranged as described above. It is shown that thirteen Intersection Areas (IAs) can be distinguished within an unknown can be localized.

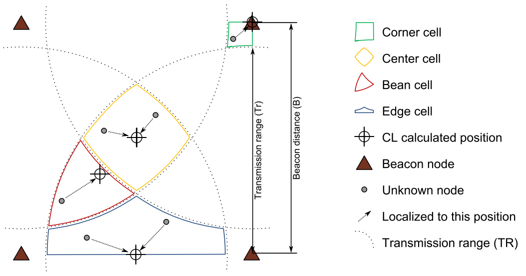

Figure 1. Basic idea of CL. Nodes assign themselves to a cell by listening to messages from the beacons. Then, the node localizes itself at the estimated centroid of the according cell.

CL uses the location information of all beacons in range to calculate the position as the centroid of the received beacon positions, as shown in equation (1). Here, $P_i(x,y)$ indicates the position of unknown i given by its two dimensional coordinates. The known position of beacon j is given by $B_j(x,y)$. The number of beacons which are within the communication range of the unknown node is indicated by m.

$$P_i(x,y) = \frac{1}{m}\sum_{j=1}^{m} B_j(x,y) \quad (1)$$

A node which is situated within one of the IAs will calculate its position at one single point, regardless of its exact position within the IA. For each IA exists such an localization point which is the centroid of the beacon positions in range. This behavior leads to a relatively high localization error, given as the Euclidian distance between the exact position of a sensor node and its calculated position.

B. Former Analyses of CL

Blumenthal et al. analyzed in [7] the average localization error as well as the maximum localization error regarding the beacon's transmission range. For evaluations, they simulated a field of 30m*30m. Within this field 16 beacon nodes have been placed on a regular grid structure. The horizontal and vertical distance between two beacons was 10m. The communication range has been varied from 0m to 42.5m, which corresponds to the range of 0% to 425% of the beacon distance. Blumenthal et al. showed that the average localization error cannot be less than 18% of the beacon distance for CL. Furthermore, the maximum localization error is 45%. Blumenthal et al. showed that the localization error depends on the ratio between the beacon distance and the communication range. The cited values belong to an optimal transmission range of 87% of the beacon distance. The low accuracy of CL has motivated development of an improved localization which is based on CL but includes techniques to reduce bias.

III. Analysis of Centroid Localization

A. Analysis of the cell areas

Former analysis of CL mostly investigated the impact of the transmission range on the accuracy of the localization. Obviously, the transmission range impacts the size of IAs and, as a result, also affects the localization error. For a detailed analysis of this relation, we classified the IAs, as depicted in figure 1, into four categories, i.e. corner cells, center cells, edge cells and bean cells. The different sizes of the cells compared to each other are illustrated in figure 2. We also depicted the standard deviation of the cell sizes, which is an indicator of the minimum difference between the cell sizes, and, therefore of the optimal transmission range. This is achieved at a transmission range of 95%. However, if we compare the real optimal transmission ratio of 87% [7] with the optimal transmission ratio estimated by the differences of the cell sizes, it becomes obvious that another error is influencing the optimal transmission range.

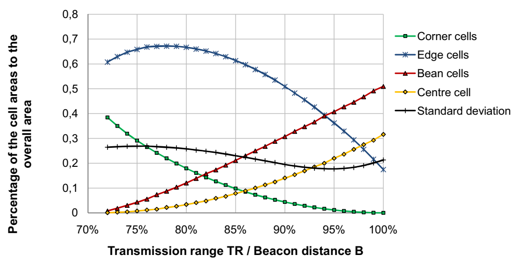

Figure 2. Fraction of different cell classes to the overall area versus the transmission ratio.

B. Analyzes of the centroid estimation

The second component which impacts the average localization error is the estimated positions of the centroid of each cell. To compare the estimated centroid's position of each cell with the real value, we simulated the arrangement of four beacons as shown in figure 1. We varied the transmission range from 72% to 100% of the beacon distance. For each of these settings we approximated the real centroid of every IA by identifying all points within an intersection and calculating the mean. As expected, the intersection built by 4 beacons (center cell) is fixed in the center of the arrangement for all settings. The centroids of all other IAs depend on the adjusted transmission range. Contrariwise, the positions calculated by CL are always the same, because CL does not take the transmission range into account. From another point of view, CL does not consider the impact of disconnected beacons. This behavior causes the bias.

The described error is illustrated in figure 3. The illustration shows the effect for IAs built by three beacons. In figure 3(A), the transmission ratio is large and the bean cells are big, but the real centroid is located nearer to the corners than the CL estimates. In figure 3(B), we set the transmission range to a low value. The emerging bean cells are relatively small and are situated near to the center. All nodes within such a cell would be localized outside the cell using the original CL. One can also find a setting where the cells centroid and the approximated CL localization coincide (figure 3(C)).

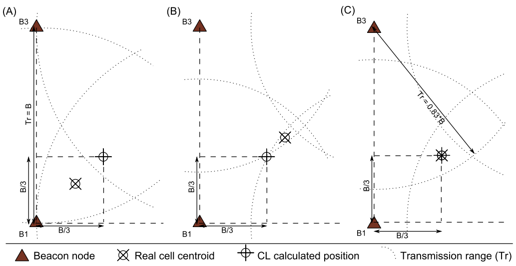

Figure 3. Centroid of the bean cell compared with the CL approximation, (A) Large transmission range, (B) Small transmission range, (C) Optimal transmission range.

Although CL constructs IAs by checking the beacons in range, the calculated centroids do not belong to the cells. CL instead calculates the centroid of a polygon built by the beacons in range. Therefore the calculated CL point is independent from the transmission range and differs from the cells centroid. This knowledge allows a further interpretation of the analysis done by Blumenthal. The accuracy of CL depends on two parameters. The first one is the number and size of the emerging cells as it was mentioned in the beginning of this section. The second parameter is the distance between the cells' centroids and the static location estimates of CL. Both parameters depend on the transmission range. The transmission range of 87% of the beacon range, which has been appointed as the optimum regarding accuracy, is obviously only the best compromise of both parameters. A better localization can be achieved by calculating the cells centroids.

C. Analyzes of edge and corner cells

Next, we consider the special cases where sensor nodes are in the range of border beacons, i.e. beacons are situated at the edge of the network. An imaginable scenario is a square room and a beacon placed in each corner. In this case, the minimum average error for edge cells and corner cells could be achieved if the assumed centroid position is chosen in the centroid of the corresponding cell, and not in the corner or the middle of the edge, like CL does, shown in figure 1. A correction of the centroids' positions would improve the basic CL algorithm for this special case, called 4-beacon scenario.

IV. Approaches to Minimize the Biased Error

As shown in the previous section, size, position and form of IAs depend on the chosen transmission range. Apparently, it is not trivial to calculate the cells centroid under the given conditions. Therefore, we present three approaches with different complexity which we jointly refer to as Shape-Based Centroid Localization SBCL.

Our first approach is a complex approximation of the real centroid. It can be regarded as a reference for the other approaches. The second one is an approximation that uses triangles, built by the points of intersection of the beacons transmission ranges. The last approach uses a linear function for constructing an approximation of the real cell centroid. All presented approaches rely on the additional precondition that the adjusted transmission range is known and constant for all nodes of the network. Also the regular arrangement of beacons as well as the beacon distance will be treated as given prior information. Our approaches will be described in detail for an IA of three beacons.

A. Exact calculation

Despite the complexity of calculating real centroids, we investigate this approach in order to characterize the ideal improvement over CL. To identify the centroids of the IAs, the whole network field (10m*10m) is divided into 10000 small rectangles, building a regular grid. For each such point it is tested which of the beacons are in range of this point. By this procedure each point becomes classified as belonging to one of the thirteen possible IAs. After that, the average position of all points belonging to the same class, i.e. with the same beacons in range, is calculated as centroid of the concerning IA. To localize an unknown node, the nodes class has to be identified and its position is set to the before calculated centroid.

B. Triangular Approximation

In order to decrease complexity, we next consider approximate calculation of centroids. As illustrated in figure 4, most of the IAs can be approximated as a triangle. Corner cells are approximated as rectangles.

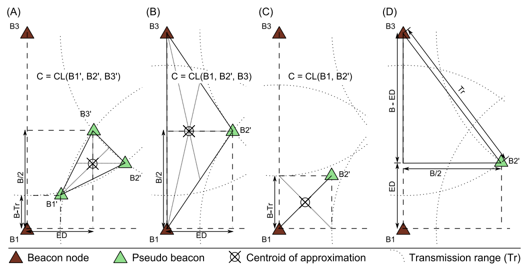

Figure 4. Triangular approximations of the pseudo beacons of (A) bean cell, (B) edge cell, (C) corner cell, (D) calculation of ED.

Knowing the vertices of triangles, the centroid can be calculated using CL. As illustrated in figure 4 this vertices will be used as pseudo beacons. In other words, the positions of the beacons in range can be substituted by the triangles vertices, i.e. by the intersections marked in the figure. One possibility to find these points is an exact calculation by solving circular equations. Considering the constraints of sensor nodes, less complex approximations to this formula are necessary. Our solution is explained with an example of the bean cell, i.e. figure 4(A). Firstly point B1' depicted in figure 4(A) can be approximated by equation (2).

$$B1' = B1 + \begin{pmatrix} B - Tr \\ B - Tr \end{pmatrix} \tag{2}$$

Secondly, one coordinate of $B2'$ (figure 4(A) and (B)) and one coordinate of $B3'$ (figure 4(A)) is on the half of the beacon distance, i.e. B/2, and can be easily estimated. The remaining parameter is the distance of B2' and B3' towards the appropriate edge, i.e. ED. As depicted in 4(D), ED can be calculated, using equation (3).

$$ED = B - \frac{1}{2}\sqrt{4Tr^2 - B^2} \tag{3}$$

Equation (4) gives an example, how $B2'$ and $B3'$ in figure 4(A) are estimated. The same estimation is used for $B2'$ in figure 4(B).

$$\begin{aligned} B2' &= B1 + \begin{pmatrix} B/2 \\ ED \end{pmatrix} \\ B3' &= B1 + \begin{pmatrix} ED \\ B/2 \end{pmatrix} \end{aligned} \tag{4}$$

Regarding figure 4(A), within a localization process a node has to decide first, in which of the four possible corners of a given arrangement of four beacons it resides. This is necessary to decide whether the calculated distances like ED have to be added or subtracted to or from a given beacon position. This can be easily realized by comparing the positions of the three beacons in range. To reduce costs for sensor nodes, calculating the intersection points can be either approximated by a linear function or calculated in advance by the beacons and transmitted with other localization data to the unknowns.

To approximate the centroid of a corner cell as depicted in figure 4(C), it is sufficient to use only two beacons. Therefore, the position of pseudo beacon $B2'$ has to be calculated in the same manner as given in equation (2).

As already mentioned, a node has to decide firstly, in which edge or corner it is located. Regarding figure 4(B) and (C), this necessitates additional knowledge. Therefore, beacons have to know there position in the network. This information can be given to the beacons during their placement.

The triangular approximations compared to the real centroids for different transmission ratios are depicted in figure 6, figure 7 and figure 8. It can be seen that the triangular approximation deliver a much better approximation for the centroids than the original CL algorithm.

C. Linear Centroid Estimation

Although the presented triangular approximation is less complex than an exact calculation of centroids, it still comprises some costly calculations. To further reduce complexity, the third approach focuses on the centroids itself. With the help of our exact approach we analyzed the distances between the correct centroid and the edges near to the controid. This edge-to-centroid distance EC is illustrated for different cells in figure 5. For the CL location point this is always a third of the beacon distance in case of a bean cell.

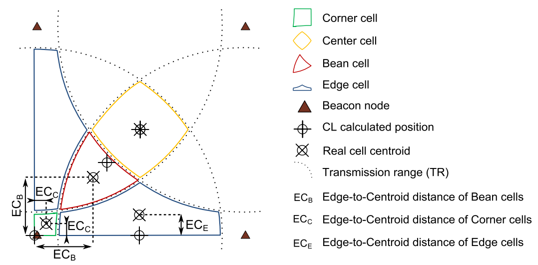

Figure 5. Illustration of edge-to-centroid distances for various cell classes.

Our aim was to find a simple function which covers the relation between transmission range and the described distance. Particularly, we found that the correlation between transmission range and edge-to-centroid distance can be approximated by using a linear function. This function, the real edge-to-centroid distance and the approximation of CL are illustrated in figure 6.

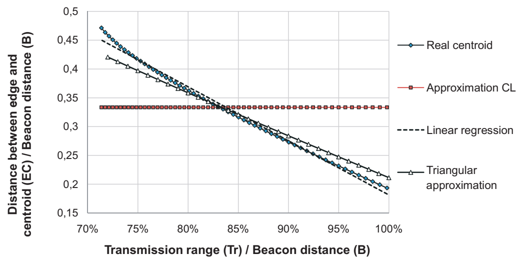

Figure 6. Comparison of estimated centroid positions of a bean cell.

Knowing this function the position will no longer be calculated by CL, but can be directly computed using this function. In contrast to our former approach, this one bypasses CL for this special case.

The same simulations have been made for corner cells and edge cells and are depicted in figure 7 and figure 8. The linear approximation, given in equation (5), enables the calculation of the approximated distances to the edges, i.e. EC, by equation (5).

$$EC = m * Tr + n * B \tag{5}$$

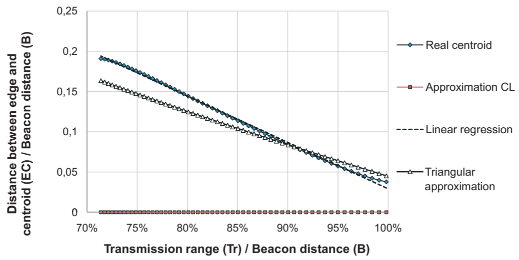

Figure 7. Comparison of estimated centroid positions of an edge cell.

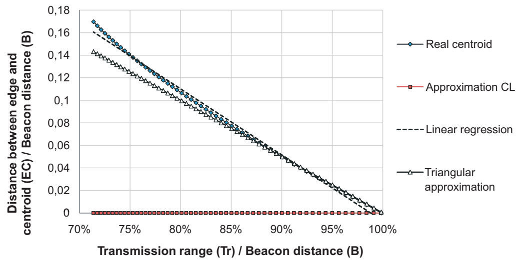

Figure 8. Comparison of estimated centroid positions of a corner cell.

For each cell type equation (5) is used with different coefficients, given in Table I.

Table I
COEFFICIENTS OF THE LINEAR OFFSET FUNCTIONS

Cell	Bean	Edge	Corner
Slope m	-0.94	-0.58	-0.59
Offset n	1.12	0.61	0.58

V. Simulation Results

For our simulations we used MATLAB. We assumed a square field of 10m*10m with 4 beacons, one in each corner. Then, we placed an unknown node on each possible position in the field with a granularity of 100*100 different positions. For each grid point, the estimated position was calculated and compared to its real one. Furthermore, average localization error and maximum localization error have been calculated as percentage of the beacon distance. We varied the transmission range from 72% to 100% of the beacon distance. A short overview, given in figure 9, shows the error distribution for CL, bean correction and correction of all cells at a transmission ratio of 90%. More detailed results are presented in the following diagrams.

The first diagram, given in figure 10, shows the average localization error of the discussed approaches. The centroid correction is only applied to bean cells. The figure shows that the usage of the centroid improves the accuracy. The presented approximations show a nearly identical performance. Furthermore, the best constellation was found with a transmission ratio of 93% and an average localization error of 16.3%.

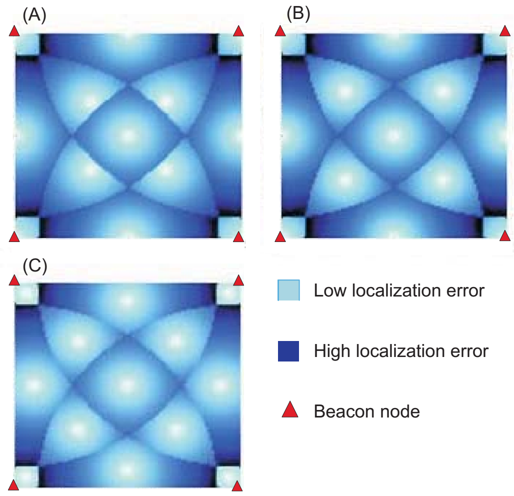

Figure 9. Error distribution of (A) original CL, (B) CL with corrected bean cells and (C) corrected bean, corner and edge cells.

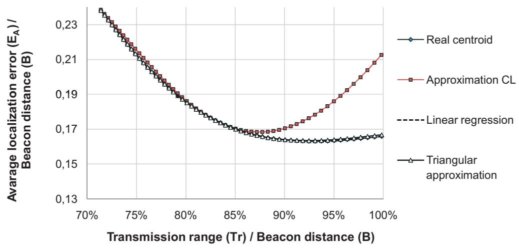

Figure 10. Average localization error of the different centroid approximation methodologies for the bean cells.

The second diagram, which is shown in figure 11 shows the achievable accuracy, if centroid correction is applied to all cell classes. This corresponds to the 4-beacon scenario. As illustrated, linear approximation performs better than triangular approximation. Accuracy of localization is significantly improved for all inspected transmission ratios.

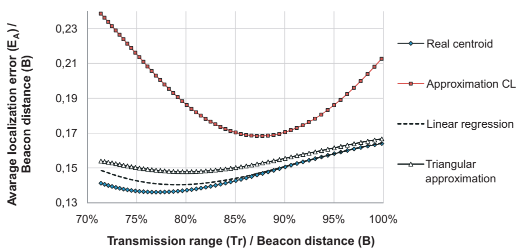

Figure 11. Average localization error of the different centroid approximation methodologies for all cells in a 4-beacon scenario.

The last diagram investigates the maximum localization error. This is shown in figure 12, where centroid correction is applied to all cell classes. The illustration shows an improvement for lower transmission ratios. Furthermore, it shows that the triangular approximation especially impacts the maximum localization error.

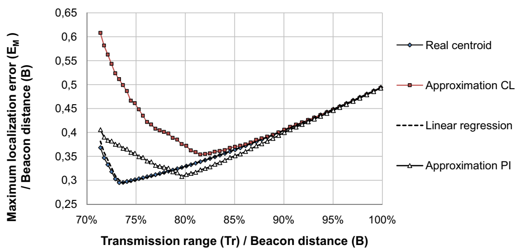

Figure 12. Maximum localization error of the different centroid approximation methodologies for all cells in a 4-beacon scenario.

VI. Conclusion and Outlook

Our SBCL approach achieves considerable improvements in accuracy by using additional topological knowledge which is inherently given. Future research may include an adaptation of other CL based localization algorithms.

The presented results showed that there is a minimum concerning both, the averaged localization error as well as the maximum localization error, for each ratio between communication range and beacon distance. This minimum can be achieved by using the real centroid of the emerging IAs instead of the points calculated by CL. Furthermore, we developed two strategies which provide accuracy close to this optimum. Regarding accuracy and computational complexity, our linear regression approximation showed a superior performance compared with the triangular approximation. Nevertheless, triangular approximation outperforms CL in terms of accuracy and is also a good solution.

Comparing both approaches, triangular approximation offers the advantage that it implicitly still uses CL for all calculations. This opens the possibility to easily adapt SBCL to other centroid based approaches like Weighted Centroid Localization (WCL), which may lead to further improvements.

We showed that correcting the centroid of bean cells as well as edge cells and corner cells leads to improved accuracy. The bean cell correction is from special significance. These cells are completely enclosed by beacons communication ranges and are not influenced by any network border. Therefore this correction can be used for regularly arranged networks with relatively large number of beacons.

Nevertheless, our approaches to correct edge cells and corner cells are also of practical importance. Indoor localization, as described in [8], often assumes environments with 4 Beacons arranged in rectangular or even square rooms. With some minor modifications, our approaches can be also used in non-square but rectangular rooms.

Acknowledgment

This work was supported by the German Research Foundation under grant number BI467/17-2 (keyword: Geosens2)

References

[1] I. F. Akyildiz, S. Weilian, Y. Sankarasubramaniam, and E. E. Cayirci, "A survey on sensor networks," *IEEE Communications Magazine*, vol. 40, no. 8, pp. 102–114, 2002.

[2] J. Salzmann, R. Behnke, M. Gag, and D. Timmermann, "4-MASCLE - Improved Coverage Aware Clustering with Self Healing Abilities," *International Symposium on Multidisciplinary Autonomous Networks and Systems (MANS 2009)*, Jul. 2009.

[3] K. Akkaya and M. F. Younis, "A survey on routing protocols for wireless sensor networks," *Ad Hoc Networks*, vol. 3, no. 3, pp. 325–349, 2005.

[4] J. Al-Karaki and A. Kamal, "Routing techniques in wireless sensor networks: a survey," *Wireless Communications, IEEE*, vol. 11, no. 6, pp. 6–28, Dec. 2004.

[5] J. D. Gibson, *The Mobile Communications Handbook*. Boca Raton FL, USA: CRC Press, 1996.

[6] N. Bulusu, J. Heidemann, and D. Estrin, "GPS-less low cost outdoor localization for very small devices," *IEEE Personal Communications Magazine*, vol. 7, no. 5, pp. 28–34, Oct. 2000.

[7] J. Blumenthal, R. Grossmann, F. Golatowski, and D. Timmermann, "Weighted Centroid Localization in Zigbee-based Sensor Networks," in *IEEE International Symposium on Intelligent Signal Processing, WISP 2007*, Madrid, Oct. 2007.

[8] S. Schuhmann, K. Herrmann, K. Rothermel, J. Blumenthal, and D. Timmermann, "Improved weighted centroid localization in smart ubiquitous environments," *Ubiquitous Intelligence and Computing*, pp. 20–34, 2008.