The LSB Procedure: Signal Analysis

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***Abstract*— Previous research has proposed the localization-by-superposing beats (LSB) procedure, which is particularly suited for low-cost indoor localization [1]. In addition to a brief review, this paper presents some laboratory results as well as a first theoretical analysis. The practical experiments were done by means of radio transmitters and receivers as well as in wired setup.**

I. Introduction

During the two decades, localization systems have been receiving increasing attention. The global positioning systems (GPS) and Galileo [2] are two well-know examples. Both systems can determinate the position of a receiver with a precision of about a few meters, which can be improved to a few centimeters under certain circumstances [3]. However, factories, laboratories, warehouse, etc., are typically *indoor* environments, which exclude GPS and Galileo to a large extent. Indoor GPS [4], [5], for example, provides a localization accuracy of about some millimeters. But due to their costs, such systems are very often not affordable in everyday life of home applications.

In general, a distance can be measured by the time-of-flight or the strength of a signal. If using electromagnetic signals for measuring the time-of-flight, a resolution in the range of a centimeter requires a sampling rate slightly higher than 3 GHz. However, with respect to a low-cost implementation and a few resource complexity, this approach is currently unfeasible. Furthermore, a resolution of about a millimeter and below will be unrealistic for a low-cost and straightforward structured devices in the near future.

For the application areas mentioned above, this paper proposes a new procedure to measures the relative distance difference of two points (e.g., transmitters) and a third point (e.g., a receiver), called localization by superposing beats (LSB). This procedure employs at least two transmitters that emit particularly modulated beats. These modulated beats lead to envelopes of low-frequency, *location-dependent* shapes. A simple detector can analyze these location-dependent shapes, and thus, reconstruct its own relative location Δx. Based on the various distance differences between more than two points well-know methods can be used to determinate the location within a 2D or 3D environment.

Section II provides a detailed description of the LSB procedure as well as some of its properties and also presents the results of a brief, preliminary theoretical analysis. Section IV presents the results that have been achieved in the laboratory environment and theirs general applicability of the LSB procedure as a localization method. The discussion in Section V describes two possible algorithms, pattern-matching an Fourier-Analysis, which derive the receiver's location Δx. Section VI then discusses the enhancement to use the procedure for multidimensional localization. Finally, SectionVII concludes this paper with a brief discussion.

II. The LSB Procedure

This section describes the localization-by-superposing-beats (LSB) procedure in detail. The presentation consists of a brief summary of some well-know preliminaries, the description of the actual procedure, as well as a discussion of some of its properties.

A. Preliminaries

The LSB procedure bases on the superposition of beats. A beat arises through the superposition of two signals $s_1(t) = \sin(2\pi f_1 t)$ and $s_2(t) = \sin(2\pi f_2 t)$ with similar frequencies $f_1 \approx f_2$. A receiver will be reading the beat $r_0(t)$:

$$\begin{aligned} r_0(t) &= s_1(t) + s_2(t) \\ r_0(t) &= \sin\left(2\pi \frac{f_1 + f_2}{2} t\right) \cos\left(2\pi \frac{f_1 - f_2}{2} t\right) \end{aligned} \quad (1)$$

The beat $r_0(t)$ (Eq. 1) is enveloped with the low-frequency signal $\cos(2\pi f_{low} t)$ with $f_{low} = (f_1 - f_2)/2$.

In this very same configuration, i.e., both transmitter remain at fixed locations, the receiver is moved by Δx, a (theoretical) phase-shift $\Delta\varphi$ occurs between the envelopes with respect to the original position. This phase-shift can be used to derive the distance Δx of the first and second receiver position. However, in order to make use of this phase-shift, the receiver would require some global timing information to calculate the phase-shift by two independent measurements.

With respect to the realization of a proper measurement system, it might be useful to note that the phase-shift can be directly derived at the *low-frequency* envelope signal.

B. The Procedure

The LSB procedure assumes the same physical setup as already described in Section II-A. But rather than emitting sinusoidal signals, the transmitters already emit beats of the form $b_i(t) = \sin(2\pi f_i t)\cos(2\pi f_m t)$ with slightly varying carrier frequencies f_i but identical envelopes '$\cos(2\pi f_m t)$'.

The major effect of *properly configured beats* $b_1(t)$ and $b_2(t)$ is that its superposition leads to an interference signal with a shape that uniquely depends on the receiver's position. Figure 1 sketches the shapes of the envelope of the interference at four distance differences Δx. The information of the relative distance difference is located in the different characteristic of the antinodes amplitudes. By evaluating the shape of this location-dependent envelope signal, a receiver is able to derive its relative distances to the two transmitters on its own.

The receiver reads the following signal:

$$\begin{aligned} r_{\Delta x}(t) &= [\sin(2\pi f_1 t)\cos(2\pi f_m t)] + \\ &\quad [\sin(2\pi f_2(t-\Delta t)) \cdot \\ &\quad \cos(2\pi f_m(t-\Delta t))] \end{aligned} \tag{2}$$

with the following parameters:

$$\Delta t = \frac{2\Delta x}{c} \tag{3}$$
$$f_1 = f_g - \Delta f \tag{4}$$
$$f_2 = f_g + \Delta f \tag{5}$$
$$f_m = \Delta f / k \ ; \ k \in Q \ ; \ k \neq 1 \tag{6}$$

In this description, Eq. (3) states that a location variation Δx affects the time-of-flight of the two beats $b_1(t)$ and $b_2(t)$ twice: a spatial shift increases the first one by $\Delta x/c$, and decreases the second one by the same amount. Eqs. (4) and (5) introduce the global carrier frequency f_g and the beat frequency Δf that both emerge from the superposition of the two carriers $f_1 \approx f_2$. Finally, Eq. (6) describes the relation between the emerging beat frequency Δf and the modulation frequency f_m. The factor k is called *pattern factor*.

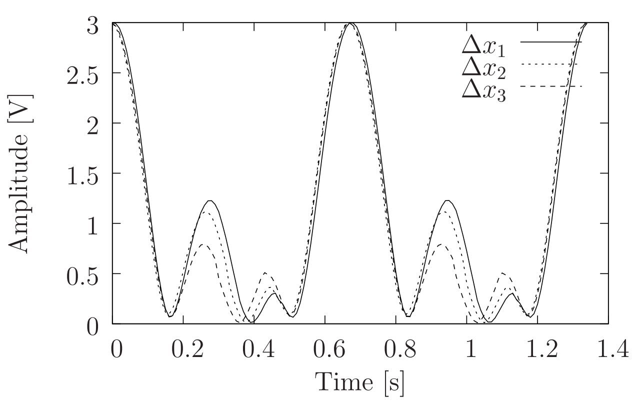

Fig. 1. Three real measured LSB periods (scaled positive envelope to 3 V). The LSB period ranges from about 0.5 s to 1.16 s. The length is equal to $1/\Delta f$ for the dedicated values of the experiment.

In order to obtain stable, location-dependent interference patterns, the quotient of these two frequencies must be a rational number, preferably an integer, e.g., $f_m = n|f_1 - f_2|$ or $nf_m = |f_1 - f_2|$, respectively, with $n \neq 1$.

C. The Envelope Fitting Function

The positive envelope of the signal $r_{\Delta x}(t)$ is approximately identically to:

$$r_{\Delta x}(t) \approx |2 \cdot \cos(2\pi f_m t)\cos(2\pi \Delta f t - 2\pi f_g \Delta t)| \tag{7}$$

provided that $f_g \gg f_m$. Eq. (7) suggest that the positive envelope can be determined by a small number of test points rather than doing a complete calculate of the function in analytic form like Eq. (2). The fitting function consequently reduces the resource complexity for detection algorithm is described in Section V.

D. Some of the Procedure's Properties

The major parameters of the LSB procedure are the carrier frequency f_g, the modulation frequency f_m and the frequency difference Δf. The resolution of the distance measurement depends on the carrier frequency f_g. If the receiver is able to detect a phase-shift of about 1°, a carrier of approximately 84 MHz would suffice to achieve a precision of 1 cm. Due to the sinusoidal nature of the carrier signals, the LSB procedure exhibits a periodic behavior. The period in which unique location-dependent envelopes emerge is defined by both the modulation frequency f_m and the frequency difference Δf. The combination of these three frequencies determines the envelope's shape, how many antinodes and nodes form the pattern, and the length of the LSB period.

The procedure's periodical behavior can be specified with the smallest distance difference with unique shapes by:

$$s = \frac{c}{2f_g}|ka - b| \tag{8}$$

The indices a and b are the solution of the extremum problem to locate the minimum of:

$$q = \left| \frac{a}{2f_m} - \frac{b}{2\Delta f} \right| \tag{9}$$

with $a, b \in N, Z$ and $1 \leq a \leq A$ and $1 \leq b \leq B$. For the factors A and B have to comply with:

$$0 = \left| \frac{A}{2f_m} - \frac{B}{2\Delta f} \right| \quad \text{with} \quad A, B \in N, Z \text{ and } \geq 1 \tag{10}$$

For a combination of $f_m = 0.5$ and $\Delta f = 1$ and $k = 2$, respectively, s ranges from $0 \leq \varphi \leq \pi$ with $\varphi = 2\pi f_g(s/c)$. For example of this combination, a frequency of 100 MHz for f_g results in an effective range of $s \approx 3\,m$. For a detailed description and discussion of the properties of the LSB procedure, the interested reader is referred to the literature [6].

E. Amplitude Discrepancy

In the above subsections, (Eq. (2)) and (Eq. (7)) have assumed that the amplitudes b_1 and b_2 match, i.e., $b_1/b_2 = 1$. Simulations show that a discrepancy causes an average recognition error of $\epsilon \leq 1\,\%$ between both amplitudes for a ratio of up to $b_1/b_2 \geq 80\,\%$. The average recognition error ϵ is the mean value over all relatively distance differences Δx in the periodical range of the LSB procedure by a increment of $\Delta\varphi = 1$ for a specific discrepancy.

The information of the location is contained in the amplitude of the received signal. Therefore, the superposition of the beats should occur with a similarity of at least $b_1/b_2 \geq 0.8$, unless the discrepancy of the two amplitudes is known; in that case, Eq. (2) can be changed adequately.

F. Influence of Noise

The LSB procedure used an adapted mode of amplitude modulation to generate the interference signal with a shape that uniquely depends on the receiver's position. The procedure also inherits the amplitude noise of the radio communication.

Another problem arises from different amplitudes: if one signal is significantly stronger than the other, then it dominates and thus corrupts the interference pattern. By contrast, noise with lower and/or higher frequencies are not a problem, since that can be compensated by properly configured band pass filters.

Discussion: The influence of non-equal signal strengths (i.e., amplitudes; see also Section II-E) can be eliminated by receiving the beats b_1 and b_2 separately, normalizing them to the same value, and superposing them within the receiver.

III. The Method's Novelity

The LSB procedure can be classified as a member of localization algorithms that measure a signal's runtime or phase shift between transmitters and receivers. One example of a method to measure the signal runtimes is the UbiSense system [7] which calculates the position of a receiver by the *time-difference-of-arrival* method (TDoA) and achieves a precision about 25 cm to 50 cm [8]. To achieve a precision of a few centimeters or millimeters, a signal frequency of more than 1 GHz as well as a system to evaluate this high-frequency-signal would be required.

The LSB procedure also uses high-frequency signals to achieve a precision of few centimeters or millimeters, but the calculation of the receiver's position is done only a low-frequency envelope signal. That is, the LSB procedure also transforms the signal analysis form a high-frequency signal to a low-frequency signal for the determination of receivers position. Furthermore, by the same configuration of the beat frequency Δf and the modulation frequency f_m, the carrier frequency f_g can be changed without the low-frequency is changed and therefore the analysis system has to be changed.

In more detail, the LSB procedure indirectly uses the signal phase shift and signal runtimes of the high-frequency transmission signals, respectively, to determinate the distance between transmitters and receivers. But the LSB procedure does not directly measured the signal phase shift or the signal runtimes of the high-frequency transmission signals.

By the pattern-matching algorithm (Sec. V-A) the shape of received signal is analyzed. The fourier-analysis algorithm admittedly determinates a phase shift but not the phase shift of the high-frequency transmission signals. With the LSB procedure, it is possible to use transmission signals with more than a few Giga-Hertz without the requirements on an evaluation device for such high-frequencies.

IV. Laboratory Experiment

A. Wire-Based

The first experiments focus on the validation of the active principle of the LSB procedure. For this case the experimental setup (Fig. 2) has been build up with a wire-based setup. With this setup, radio transmission problems, such as multipath scattering and attenuation of signals, could be easy avoided and the requirements for a wire-based setup is also less then for radio transmission.

The signals of the two transmitters, represented by two line-stretchers (Microlab SR-05F), are generated by the Analog Devices DDS AD9959 evaluation board. The beats of the transmission signals are generated by the mini-circuit combiner ZFSC-2-1. Also the receiver is substituted by a combiner. By the two line-stretchers, the moving of the receiver has been simulated. From the received interference shape, the envelope was detected by the Analog Devices TruPwr™ Detection RFIC chip AD8361 and then discretized by analog-to-digital converter, which is directly connected to a PC for recording the envelope and determination of the location Δx with both above described detection algorithm. The receiver has always been positioned in a fictitious straight line in between the two transmitters.

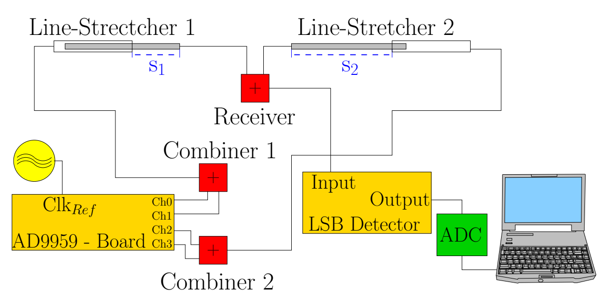

Fig. 2. Schematic diagram of the laboratory setup. All wire-pairs have the same lengths between the AD9959-Board and the Receiver for the both transmissions lines.

In the experiment, the receiver has been moved by $s_1 = s_0 \pm \Delta x$ and $s_2 = s_0 \mp \Delta x$ at the same time in a range of $0 \leq \Delta x \leq 15\,\text{cm}$ by steps of 1 cm. For all experiments, three carrier frequencies of f_g = [50, 100, 150] MHz and the following factors of k = [0.25; 0.5; 2; 4] by $\Delta f = 2.98$ Hz have been used.

In summery of the results of the effected experiments it can be specified for the LSB procedure a maximum error of ±1 cm. If the results are projected onto the LSB procedure in general it can be specified a maximum error of $\Delta\varphi \approx \pm 2°$ for the used carrier frequency f_g

B. Radio-Based

In the second set of experiments was using radio signals The two line-stretchers were replaced by two 50 MHz custom-made transmitters and the receiver combiner was replaced by a 50 MHz custom-made radio receiver (Fig. 3). The two transmitter was arranged in a direct line of sight with a distance of 2 m. The receiver was moved in steps of 10 cm between the two transmitters. In these experiments, only the carrier frequency $f_g = 50$ MHz and the factor $k = 2$ by $\Delta f = 2.98$ Hz have been used.

The chosen experimental setup was simple without taking any precaution for multipath scattering. In summary, the results suggest that the LSB procedure can also operate with wireless communication modules. However, these preliminary experiments do not indicate the procedure's limits with respect to precision and maximal errors.

V. Detection Algorithm

A. Pattern-Matching

Both the description and experiments presented above have shown that in its operation, the LSB procedure is dependent on the amplitudes of the two receiving beats (which both superpose in the air due to physical reasons). In order to observe a unique shape of the resulting envelope, it is useful that at the receiver's side, both signals are of approximately the same strength. With respect to this "constraint", a detector might work as follows:

- For a set of fixed parameters f_g, Δf and f_{mod}, the detector pre-calculates a collection of patterns for a variety of locations Δx that correspond to $\Delta\varphi = 2\pi f_g \Delta t$ with, for example, $\Delta\varphi = [0°, 1°, 2°, \ldots, 360°)$. These patters are normalized between zero and one.
- The receiver extracts one period of the received envelope and as well scales the data between zero and one.
- For every pre-calculated pattern, the detector calculates the correlation value K_m between the signal and each pattern m:

$$K_m = \sum_{i=1}^{N} (y_{\text{signal}}(i) - y_{\text{pattern}}(i))^2 \tag{11}$$

- Finally, the lowest correlation value K_m indicates the best matching pattern, which represents the receiver's location Δx.

B. Fourier-Analysis

A second method for the determination of the receiver's location is based on the Fourier analysis and its phase-spectrum. The phase $2\pi f_g \Delta t$ can be calculated by the Fourier-Analysis from the envelope fitting function (Eq. (7)). If the received signal $r(t)$ is multiplied by the transformation function:

$$w(t) = \text{sgn}(\cos(\omega_m t)) = \begin{cases} 1 & \omega_m t > 0 \\ 0 & \omega_m t = 0 \\ -1 & \omega_m t < 0 \end{cases} \tag{12}$$

with a phase relation between the received signal $r(t)$ and the transformation function $w(t)$ so as the phase φ_{f_m} of the modulation frequency f_m spectral line is zero or a minimum for real systems, respectively, the phase spectrum has spectral lines of multiple of the beat frequency Δf on the position (δ-Function):

$$\begin{aligned} \phi_p \quad = \quad & 2p\varphi \cdot [\delta(2\pi 2p\Delta f - 2\pi f_m) + \\ & \delta(2\pi 2p\Delta f + 2\pi f_m)] \end{aligned} \tag{13}$$

with $p = 0, \pm 1, \pm 2, ...$ Equation (13) can be derived from the Fouriers properties time shift and convolution in the frequency domain.

$$h(t - t_0) \quad \circ\!\!-\!\!\bullet \quad H(f)e^{-j2\pi f t_0} \quad \text{Time Shift} \tag{14}$$

$$h(t) \cdot g(t) \quad \circ\!\!-\!\!\bullet \quad H(f) * G(f) \quad \text{Convolution} \tag{15}$$

In a subsequent process, such as correlation, the minimum can be determined from the phase spectral line for the modulation frequency f_m. The phase spectrum of the transformed signal $\tilde{r}(t) = r(t) \cdot w(t)$ is shown in Figure 4). It is conspicuously identifiable two pairwise phase spectral lines for the phases ϕ_p by the indexes p = [1,2] by the phases $2p\Delta f - f_m$ and $2p\Delta f + f_m$. One or both of the pairwise phases can be used to calculate the receiver's location Δx.

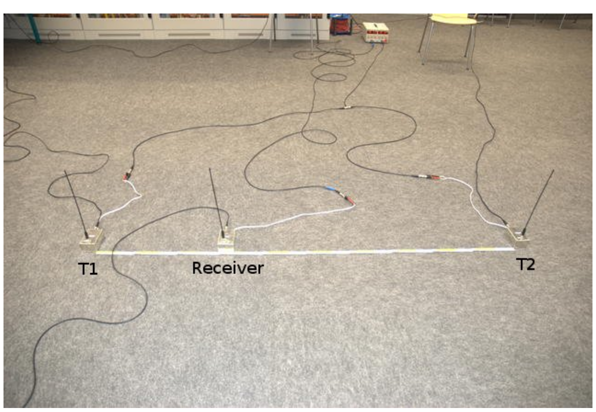

Fig. 3. Radio-based setup with the two transmitters (T1 and T2) and the receiver between them.

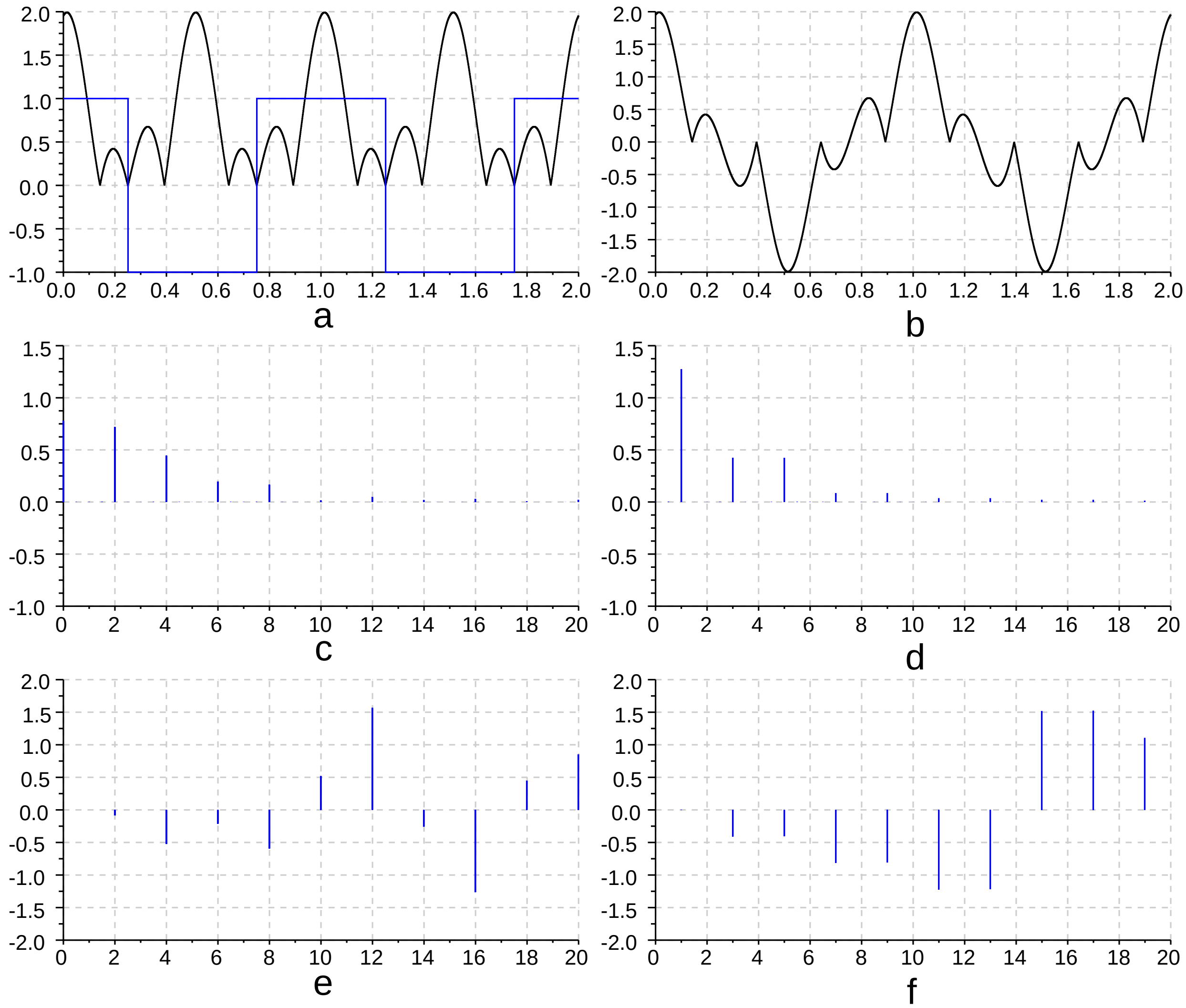

Fig. 4. The received signal $r_{\Delta x}(t)$ [a] and the transformed signal $\tilde{r}_{\Delta x}(t)$ [b] with amplitude [c/d] and phase spectrum [e/f] each plus the transformation function $w(t)$ [a (rectangle curve)]. The plotted example is created by the parameters $\Delta f = 2\,\text{Hz}$, $f_m = 1\,\text{Hz}$, $f_g = 100\,\text{MHz}$ and a receiver location of $\Delta x = 10\,\text{cm}$.

VI. Multidimensional Localization

For a real-world localization system it is essential that localization takes place in at least two dimensions if not three. To this end, the LSB system can be extended to three or four transmitters, which have all different carrier frequencies. Depending on the local arrangement, all transmitters might employ the same modulation frequency f_m. As an alternative, two or three one-dimensional LSB system might be put in place. These systems can then have entirely different configurations. In any case, a receiver can derive two (three) relative distances Δx_i, from which it can calculate its own position.

VII. Conclusion

The laboratory experiments have demonstrated the principle utility of the LSB procedure for measuring the relative distance difference between two points (transmitters) and a third point (receiver). The results of the measurements show that it is possible to measure the relative distance differences by a very low-cost architecture at an acceptable maximum error for indoor localization, particularly if higher frequencies are used. The next experimental step is to improve the wireless setup to reading the superposing beats by antenna.

It should be noted that in its current version, the LSB procedure does not address the problem of multi-path propagation, which might occur in real-world scenarios.

In its current version, the method constitutes a distance measurement system for one dimension. To upgrade the method to two dimensions, for example, the setup could be extended by two transmitters. For each transmitter combination the receiver alternately determines the location Δx

and calculates the two dimension position.

The LSB procedure addresses also a method to measure the relative distance between to transmitters with the possibility of a precision of a few centimeters in dependency on the high-frequency carrier frequency but with analysis of a low-frequency signal, which can be sampled with a sampling rate of few kilo-hertz. The receiver determines its location itself without additional synchronization or other references. Both transmitter and receiver can be build up with simple low-cost technology components. Hereby a low-cost localization system with a precision of a few centimeters can be realized.

Both, stable signal transmission by radio signals, including the multipath scattering, and to upgrade the method to two dimension will be dedicated to future research.

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