Improved Precision of Coarse Grained Localization in Wireless Sensor Networks

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Abstract

In wireless sensor networks, the coarse grained localization is a method to compute the position of randomly distributed sensor nodes. Without optimizations, it provides low precision which heavily depends on the transmission range of base stations. In this paper, we propose novel optimizations of Coarse Grained Localization with Centroid Determination (CGLCD) to determine the position of nodes more precisely. Our focus is to compute an optimal transmission range of all base stations and to reduce the total energy consumption. We present an analytic proof of a simple equation to determine the optimal transmission range in grid-aligned finite wireless sensor networks. Using this optimal transmission range, we reduced the positioning error about 80%. Thereby, nodes as well as base stations require lowest energy.

1. Introduction

Sensor networks consist of hundreds or thousands of tiny sensor nodes that are randomly spread out over an area of interest. The objective is to measure conditions of the environment and propagate this information to data sinks of the system. Further fundamentals of wireless sensor networks are described very comprehensively in [1].

A sensor node typically consists of a battery, a microprocessor, a communication module, sensors, and/or actuators. Minimizing the node to the desired size of a few millimeters is limited by the dimensions of the communication module and the battery. Due to the small size of the battery, the most limited resource within a network is the available energy. Therefore besides power saving hardware components, low power optimized algorithms are required.

Simple uncoordinated seeding of nodes yields a stochastic distribution of nodes after deployment phase. This distribution inhibits the assignment of a measured value to its location. Therefore, a position determination of all nodes is necessary which consumes additional energy for calculations and data transmissions.

This paper is subdivided as follows. In Section 2, we classify positioning algorithms. Next, in Section 3, we explain the Coarse Grained Localization with Centroid Determination. Then, we describe two boundary conditions for our analytical proof in Section 4 to apply this algorithm optimally concerning positioning accuracy and power constraints. Section 5 deduces our analytical proof to obtain the proved precision localization in finite sensor networks and presents a simple equation for calculating the optimal transmission range of beacons. Next in Section 6, we discuss our proved equations and explain practical applications. Finally, we conclude the paper in Section 7.

2. Preliminaries

Positioning algorithms are classified into coarse and fine-grained localization. Fine-grained localization facilitates high precision of position determination but results in extensive calculations and high network traffic [2]. However, exact positions are not always required. These algorithms merge low calculation requirements with less network traffic. Different coarse approaches were presented such as the Coarse Grained Localization with Centroid Determination (CGLCD) [3], exploitation of geometric constraints between the nodes [4], to narrow down the area [5], and to compute relative positions in a global coordinate system [6].

Additionally, fine-grained localization requires precise distance measurements. The precision of the

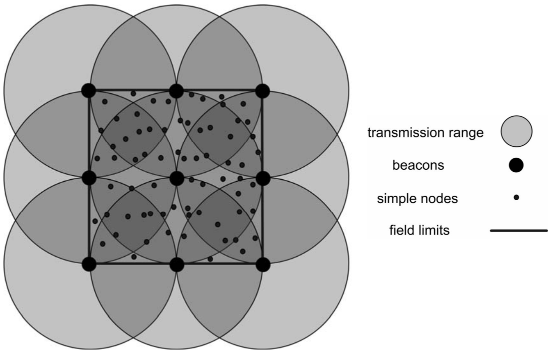

Figure 1. Overlapping regions in a 3×3 array with 9 beacons and 68 randomly distributed sensor nodes.

position determination highly depends on the accuracy of these distances. Distances can be acquired for instance by measuring the Received Signal Strength Indicator (RSSI) in the receiver. The RSSI decreases quadratically with the range to the transmitter and; therefore, represents a distance. Further to reduce costs, all sensor nodes in large networks consist of low-cost hardware measuring not very precisely.

We concentrate on coarse grained localization without measuring distances. The main focus of this paper is to optimize the CGLCD-Algorithm. We present an analytical method to determine optimized parameters such as transmission range of base stations (further on defined as beacons) and positioning error prior to distribution of nodes.

Different research groups investigated the finding of an optimal transmission range. Deng et al. proposed in [7] an analytical model to investigate the optimal transmission range based on the underlying device energy consumption model and a two-dimensional Poisson node distribution. Furthermore, Zuniga and Krishnamachari analyzed in [8] the impact of the transmission radius by flooding packets. It was showed for large wireless networks that there exists a transmission range that minimizes the time at which all nodes in the network finish transmitting. In contrast to these valuable solutions, we finally obtain in this paper a very simple equation that is analytically proved for finite networks.

3. Background: Coarse Grained Localization with Centroid Determination

In 1989, an algorithm of coarse grained localization was successfully implemented in the "Active Badge System" for indoor use based on infrared technique [9]. Later, the Coarse Grained Localization with Centroid Determination was proposed by Bulusu [3]. This algorithm is the basis of our parameter optimization.

A two-dimensional finite sensor network using CGLCD consists of l sensor nodes and b beacons. Beacons are sensor nodes knowing their own position $(x_1 ... x_b);(y_1 ... y_b)$ within the network. The beacons $B_1 ... B_b$ are deployed in a grid-aligned network (infrastructure case) with constant distance d to each other. Fig. 1 demonstrates an example of a sensor network with 3×3 beacons, where beacons form a quadratic array with randomly distributed sensor nodes inside. Prior to localization, all sensor nodes do not know their own position.

During positioning, beacons periodically send broadcast messages containing their own position. All sensor nodes within transmission range of beacons receive these messages. The messages are assumed to be sent without interfering each other to avoid collisions and packet loss. Every node observes the transmission medium for a defined period Δt registering all received messages. Then, the node determines the number of beacons n in range. Nodes that do not have beacons in range are not able to determine their own position and therefore are called unknowns.

CGLCD uses an idealized radio model to determine the nodes' positions. Perfect circular radio waves and thus identical transmission ranges r for all beacons are assumed. Thereby, it is possible to divide the array geometrically into overlapping regions. Depending on the distribution of beacons, regions with $0 ... b$ overlaps are formed (Fig. 1). Each sensor node within an overlapping area approximates its position based on the received beacon coordinates. The approximated position x_{iapp}, y_{iapp} is the centroid of the formed overlap as described in (1).

$$x_{i_{app}}, y_{i_{app}} = \left(\frac{1}{n} \sum_{k=1}^{n} x_{B_k}, \frac{1}{n} \sum_{k=1}^{n} y_{B_k} \right) \qquad (1)$$

x_{B_k}, y_{B_k} = *Position of beacon* B_k

n = *Number of beacons in range*

Each sensor node estimates its position with an approximation error $f_i(x,y)$. This positioning error is defined as distance between the approximated and the exact position of the node as represented:

$$f_i(x,y) = \sqrt{(x_{i_{app}} - x_{i_a})^2 + (y_{i_{app}} - y_{i_a})^2} \quad (2)$$

$x_{i_{app}}, y_{i_{app}}$ = *Approximated coordinates*

x_{i_a}, y_{i_a} = *Exact coordinates*

Fig. 2 depicts the relative positioning error over an array of 3×3 beacons with 100×100 nodes. Notice, the error boosts with an increasing distance to the centroid position x_{iapp}, y_{iapp} and is maximal in the vicinity of beacons. Further, the analytical function $f_i(x,y)$ is discontinuous and behaves continuous only within an overlap.

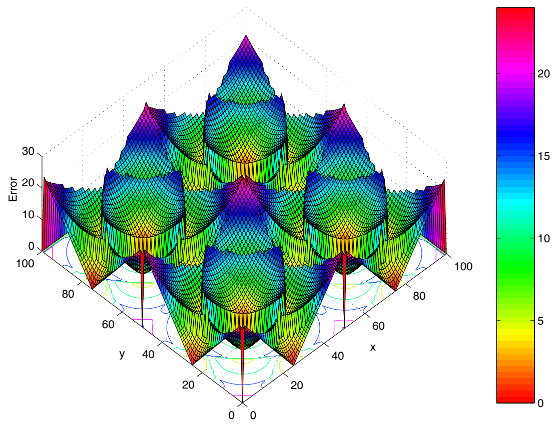

Figure 2. Error distribution over an array of 3×3 beacons, transmission range=50, array width=100. The beacons are arranged as in Fig. 1.

4. Boundary Conditions in Finite Sensor Networks

The positioning error $f_i(x,y)$ is defined as distance between approximated and exact position (2). According to (1), the approximated position of a node and therefore its positioning error depends on the number n of received position information of beacons. This number increases with a raising transmission range of the beacons. Thus, a raising transmission range considerably affects the positioning error.

Fig. 3 illustrates three sensor network scenarios using different transmission ranges with 5×5 beacons. Fig. 3a demonstrates a worst-case scenario with a very low transmission range. It is impossible for most of the sensor nodes to receive messages from beacons. Thus, they are in no overlapping region and cannot determine their position – they are unknowns. Complementary, a very high range as visualized in Fig. 3c is useless, because all nodes are within only one overlapping region. Consequently, all nodes determine their own position in the centroid of the whole array. In both cases, the average error reaches a maximum. Thus, an optimal transmission range r_{opt} exists as exemplarily represented in Fig. 3b. In this scenario, all sensor nodes compute their position based on information received only from direct neighboring beacons. Moreover, a homogeneous error distribution is achieved.

Before determining the optimal transmission range, two boundary conditions r_{min} and r_{max} are introduced. The lower border r_{min} indicates the transmission range where all sensor nodes receive at least one beacon information and; therefore, are able to determine their own position. Hence, r_{min} is equal to the half diagonal of a square enclosed by four beacons (Fig. 3b). At this transmission range, the precision is very low.

In infinite sensor networks, an increasing transmission range of the beacons results in a higher precision of the positioning due to smaller overlaps. However, in practice sensor networks are finite. In grid-aligned finite sensor networks, huge transmission ranges provoke scenarios as demonstrated in Fig. 3c. In particular, these scenarios are caused by "unbalanced" overlaps in the borderland of the network. In an unbalanced overlap, the centroid is unequal to the center of the overlap. Moreover, the centroid is shifted to the middle of the network due to missing outside beacon information. Therefore, the positioning error of nodes increases in these areas and generates a non-uniform error distribution. Hence in finite sensor networks, the average positioning error increases with the transmission range. In other words, small average positioning errors and a uniform error distribution demand balanced overlaps without shifted centroids. Balanced overlaps only exist if $r<d$. Therefore, r_{max} must be lower than distance d.

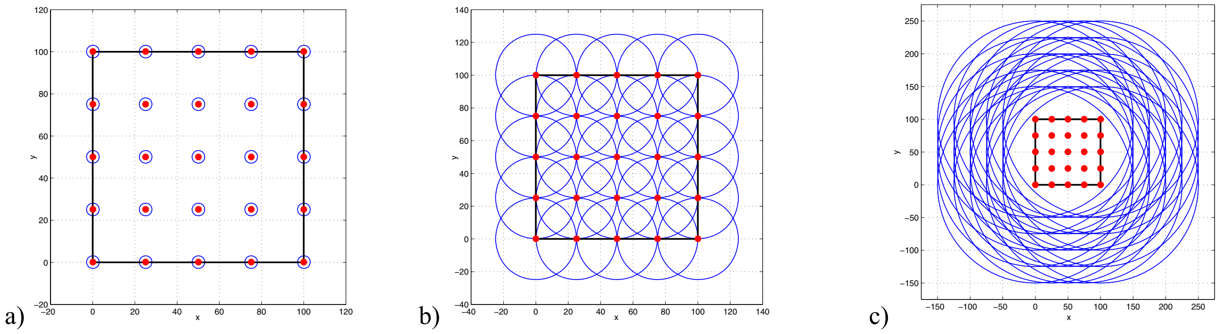

Figure 3. Three scenarios showing a finite wireless sensor network within the solid rectangles consisting of randomly distributed sensor nodes (not shown) and 5×5 beacons with a) low, b) optimal, c) high transmission ranges. In all cases, sensor nodes determine their own position based on positioning information received from neighboring beacons.

In addition besides the precision, low energy consumption is demanded. In fact, huge transmission ranges increase the precision in huge sensor networks, but they are inefficient due to their high energy consumption. As a consequence, a trade-off between transmission range, energy consumption, and position precision is required. An adequate performance criterion is the Power-Error-Product (PEP), which was introduced in [10]. Besides the average positioning error, PEP considers the energy consumption of a transmission. In vacuum with ideal environmental conditions, the energy quadratically increases with the transmission range r as denoted in (3).

$$PEP = r^2 \cdot \frac{\sum_{i=1}^{l} f_i}{l} \tag{3}$$

l	=	*Number of nodes in the sensor network*
r	=	*Transmission range*
$f_{l_{mean}}$	=	*Averaged positioning error in the sensor network*

In reality, the energy consumption even raises approximately with the power of four due to obstacles and interferences.

5. Improved Precision in Finite Sensor Networks

This Section describes the derivation and the proof of an equation to determine the optimal transmission range. The proof applies to finite sensor networks with boundary conditions defined in Section 4.

In the infrastructure case, identical overlaps within the whole array are formed as represented in Fig. 4. Therefore, further considerations are only performed for one sub-array A enclosed by four beacons. These beacons are arranged in a square with distance d to each other (Fig. 4). All beacons feature the same transmission range r. According to Fig. 5, a sub-array A consists of 16 regions A_x. These regions contain different boundaries and centroids. Due to homogeneity of some regions, the considered regions are simplified to four basic areas ($4{\cdot}A_1$, $4{\cdot}A_2$, $8{\cdot}A_3$, $4{\cdot}A_4$).The distance of a certain point in a region to the centroid of the same region is defined as positioning error $f_i(x,y)$ as mentioned in (2). Considering one region, the integration of all positioning errors results in a cumulated region error E_x. The addition of all region errors $E_1 \dots E_4$ forms the overall error E_A of a sub-array A as depicted in (4). To determine the optimal transmission range r_{opt}, the overall error E_A has to be determined and minimized depending on r.

Each of the four regions has a unique centroid C_x, which represents the approximated nodes' positions as described in (1). Therefore, $f_i(x,y)$ differs in these regions. Hence, $E_A(r,d)$ is a discontinuous function. Thus, $E_1 \dots E_4$ have to be solved separately. Below, we present a solution to determine the cumulated error $E_2(r,d)$ of region A_2. The remaining regions A_1, A_3, and A_4 are solved similarly.

$$\begin{aligned} E_A(r,d) &= \iint_A f_i(x,y)\,dA \\ E_A(r,d) &= \iint_A \sqrt{(x_{app}-x_a)^2 + (y_{app}-y_a)^2}\,dA \rightarrow \min. \end{aligned} \tag{4}$$

Centroid Determination: The centroid C_2 of the overlapping region A_2 is determined by the position information received from beacons B_2, B_3, and B_4 (see Fig. 5) in (5).

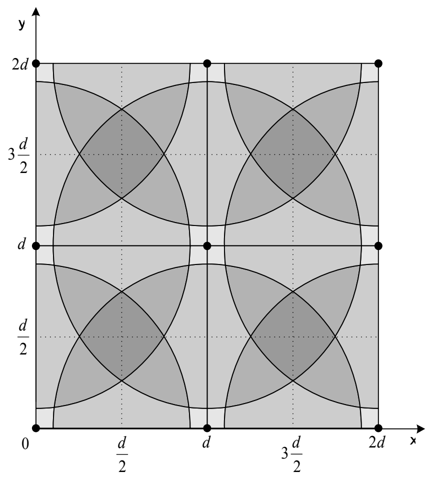

Figure 4. Four overlapping regions in an array of 3×3 beacons with transmission range $r_{\min}<r<r_{\max}$

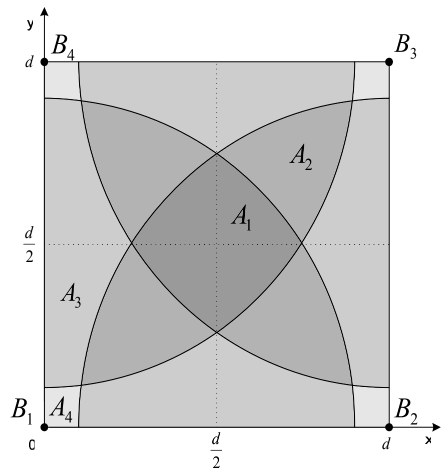

Figure 5. One overlapping region *A* enclosed by beacons $B_1 \ldots B_4$ and subdivided into regions ($4 \cdot A_1$, $4 \cdot A_2$, $8 \cdot A_3$, $4 \cdot A_4$)

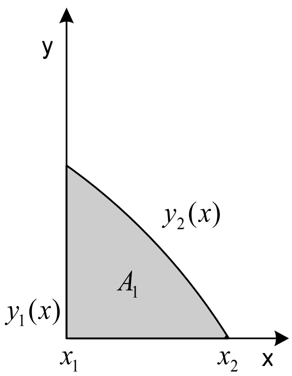

Figure 6.

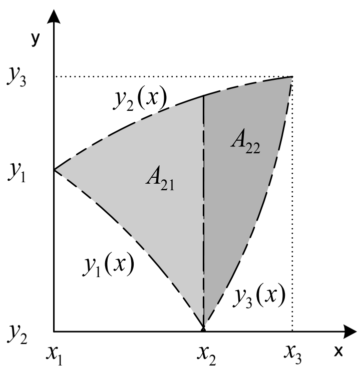

Figure 7.

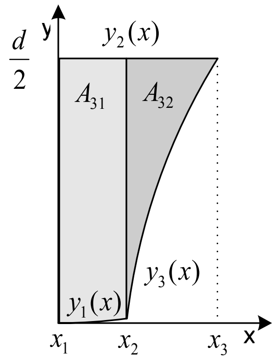

Figure 8.

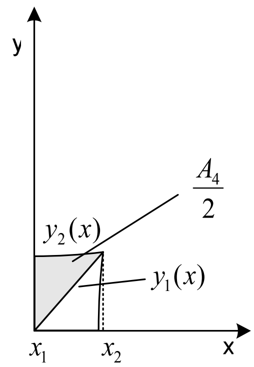

Figure 9.

Overlapping region of partial area A_1, A_2, A_3, A_4

$$C_2 = \left(\frac{X_{B_2} + X_{B_3} + X_{B_4}}{3}, \frac{Y_{B_2} + Y_{B_3} + Y_{B_4}}{3} \right) = \left(\frac{d+d+0}{3}, \frac{0+d+d}{3} \right) = \left(\frac{2}{3}d, \frac{2}{3}d \right) \quad (5)$$

Error Function: The error function $f_2(x,y)$ represents all positioning errors within overlapping region A_2. Based on (2), the error is equal to the distance between C_2 and the exact position of any node within A_2.

$$f_2(x,y) = \sqrt{\left(x - \frac{2}{3}d \right)^2 + \left(y - \frac{2}{3}d \right)^2} \quad (6)$$

Integration Limits: $E_2(r,d)$ is determined by the integration over A_2 as visualized in Fig. 7. The surface integral over A_2 is separated in an inner and outer integral. The outer integration limits x_1 and x_3 are constant. The inner integration depends on bordering functions $y_1..y_3(x)$. These limits are not monotonous at $P_2(x_2,y_2)$. Hence, the overlapping region A_2 is subdivided into two regions A_{21} and A_{22}.

Thus, (4) has to be solved for A_{21} and A_{22} separately. The inner limits of A_{21} and A_{22} are equal to the function $y_1(x)$ and $y_2(x)$ as well as $y_3(x)$ and $y_2(x)$, respectively. The outer limits are enclosed simply by x_1, x_2, and x_3.

The points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are always on one of the coordinate axes. According to Fig. 5, x_1 and y_2 are equal to the half distance d, (7) and (9). To obtain the right integration limit x_2 of A_{21}, a circle equation is formed centered at $B_1(0,0)$ as represented in (8). The equation is solved at $P_2(x_2,y_2)$ and results in (10).

$$x_1 = \frac{d}{2} \tag{7}$$

$$r^2 = x^2 + y^2 \tag{8}$$

$$y = \frac{d}{2} \tag{9}$$

$$x_2 = +\sqrt{r^2 - \frac{d^2}{4}} \tag{10}$$

Further, the circles of beacons $B_2(d,0)$ and $B_4(0,d)$ are described by (11) and (12). The circles intersect at $P_3(x_3,y_3)$ as illustrated in Fig. 7.

$$r^2 = (x-d)^2 + y^2 \tag{11}$$

$$r^2 = x^2 + (y-d)^2 \tag{12}$$

To determine the right integration limit x_3, (11) is rearranged to y and inserted into (12). The correct solution at $P_3(x_3,y_3)$ of the resulting quadratic equation is visualized in (13).

$$x_3 = \frac{d}{2} + \sqrt{\frac{2r^2 - d^2}{4}} \tag{13}$$

Based on the circle equation of beacon B_1 as manifested in (8), the integration limit $y_1(x)$ is simply formed by rearranging. The other integration limits $y_2(x)$ and $y_3(x)$ are identified using circle (11) and (12).

$$\begin{aligned} y_1 &= +\sqrt{r^2 - x^2} \\ y_2 &= +\sqrt{r^2 - (x-d)^2} \\ y_3 &= -\sqrt{r^2 - x^2} + d \end{aligned} \tag{14}$$

Integration: The cumulated positioning E_2 of region A_2 is determined by adding the positioning errors E_{21} and E_{22}, see (15).

$$\begin{aligned} &= \iint_A f_2(x,y)\,dA = \iint_{A_{21}} f_2(x,y)\,dA + \iint_{A_{22}} f_2(x,y)\,dA \\ &= \int_{x_1}^{x_2}\int_{y_1(x)}^{y_2(x)} f_2(x,y)\,dy\,dx + \int_{x_2}^{x_3}\int_{y_3(x)}^{y_2(x)} f_2(x,y)\,dy\,dx \\ &= \left| \int_{\frac{d}{2}}^{+\sqrt{r^2-\frac{d^2}{4}}} \int_{+\sqrt{r^2-x^2}}^{+\sqrt{r^2-(x-d)^2}} \sqrt{\left(x-\frac{2}{3}d\right)^2 + \left(y-\frac{2}{3}d\right)^2}\,dy\,dx \right| + \\ &\quad \left| \int_{+\sqrt{r^2-\frac{d^2}{4}}}^{\frac{d}{2}+\sqrt{\frac{2r^2-d^2}{4}}} \int_{d-\sqrt{r^2-x^2}}^{+\sqrt{r^2-(x-d)^2}} \sqrt{\left(x-\frac{2}{3}d\right)^2 + \left(y-\frac{2}{3}d\right)^2}\,dy\,dx \right| \end{aligned} \tag{15}$$

Based on the overlapping regions displayed in Fig. 6,8,9, the cumulated positioning errors E_1, E_2, and E_3 of the regions A_1, A_3, and A_4 are determined as presented in (16).

$$\begin{aligned} E_1(r,d) &= \left| \int_{\frac{d}{2}}^{+\sqrt{r^2-\frac{d^2}{4}}} \int_{\frac{d}{2}}^{+\sqrt{r^2-x^2}} \sqrt{\left(x-\frac{d}{2}\right)^2 + \left(y-\frac{d}{2}\right)^2}\,dy\,dx \right| \\ E_3(r,d) &= \left| \int_0^{\frac{d}{2}-\frac{\sqrt{2r^2-d^2}}{2}} \int_{d-\sqrt{r^2-x^2}}^{\frac{d}{2}} \sqrt{x^2 + \left(y-\frac{d}{2}\right)^2}\,dy\,dx \right| + \\ &\quad \left| \int_{\frac{d}{2}-\frac{\sqrt{2r^2-d^2}}{2}}^{d-\sqrt{r^2-\frac{d^2}{4}}} \int_{+\sqrt{r^2-(x-d)^2}}^{\frac{d}{2}} \sqrt{x^2 + \left(y-\frac{d}{2}\right)^2}\,dy\,dx \right| \\ E_4(r,d) &= 2\left| \int_0^{\frac{d}{2}-\frac{\sqrt{2r^2-d^2}}{2}} \int_x^{d-\sqrt{r^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx \right| \end{aligned} \tag{16}$$

The sum of all four integrations $E_1 \ldots E_4$ forms (17) to compute the cumulated positioning error $E_A(r,d)$ of a sub-array A.

$$\begin{aligned} E_A(r,d) = 4E_1(r,d) + 4E_2(r,d) + \\ 8E_3(r,d) + 4E_4(r,d) \end{aligned} \tag{17}$$

To obtain the optimal transmission range r_{opt}, the minimum of $E_A(r,d)$ is required. Thus, $E_A(r,d)$ is differentiated by dr at a constant distance d as demonstrated in (18).

$$\left.\frac{dE_A(r,d)}{dr}\right|_{d=const} = 0 \rightarrow r_{opt} \qquad (18)$$

In the geometry depicted in Fig. 5, only one optimal transmission range r_{opt} exists within the derived boundary conditions $r_{min}<r_{opt}<r_{max}$. This optimal transmission range r_{opt} and the distance d form the relationship G_{opt} as demonstrated in (19).

A scaling of this geometry affects distance d as well as optimal transmission range r_{opt} similarly. Hence, G_{opt} remains to be constant for this topology at all.

$$G_{opt} = \frac{r_{opt}}{d} \qquad (19)$$

Due to missing explicit solutions of (17) and (18), a numerical Matlab simulation yields G_{opt}≈0.86.

$$r_{opt} = 0.86d \qquad (20)$$

In practice, mostly distance d between beacons is predetermined and r_{opt} is demanded. Since G_{opt} is constant, (19) is reformed and results in (20) to determine the optimal transmission range r_{opt} easily.

6. Discussion

We verified the theoretical assertions in simulations with Matlab 6.5.0 R13. The dimensions of the grid array are 100×100 in all simulations. We calculated the positioning error for 10000 (100×100) sensor nodes and determined the average positioning error of all nodes in the sensor network.

In the first simulation, the correlation of transmission range r, the number of beacons and the positioning error is examined basing on the original algorithm without using analytic equations (Fig. 10a). Obviously, starting with a high value, the positioning error quickly drops to a minimum by increasing the transmission range. Then, the error increases nearly linear with the transmission range. However, the objective is a high precision with few beacons and, if possible, a small transmission range. This leads to the determination of the optimal range. As shown in Fig. 10a, the optimal transmission range decreases with an increasing number of beacons and reduces the positioning error at 10×10 beacons to 1.89%. These considerations result in an optimization problem between number of beacons vs. positioning error, and transmission range.

Fig. 10a illustrates the error behavior caused by an increasing transmission range. Generally, oversizing the transmission range is unfavorable and leads to an increased error and, additionally, energy waste. Using only four beacons, the noticeable shape of the shown curve proves an unfavorable relationship between transmission range, beacons, and size of array. Thus, too few beacons are not recommended. As one can see, the positioning error varies and heavily depends on the transmission range. With careful optimization of parameters, the error is reduced from 35 to 5 equal to savings of 80%.

In every simulation, the lowest positioning error is

a) b)

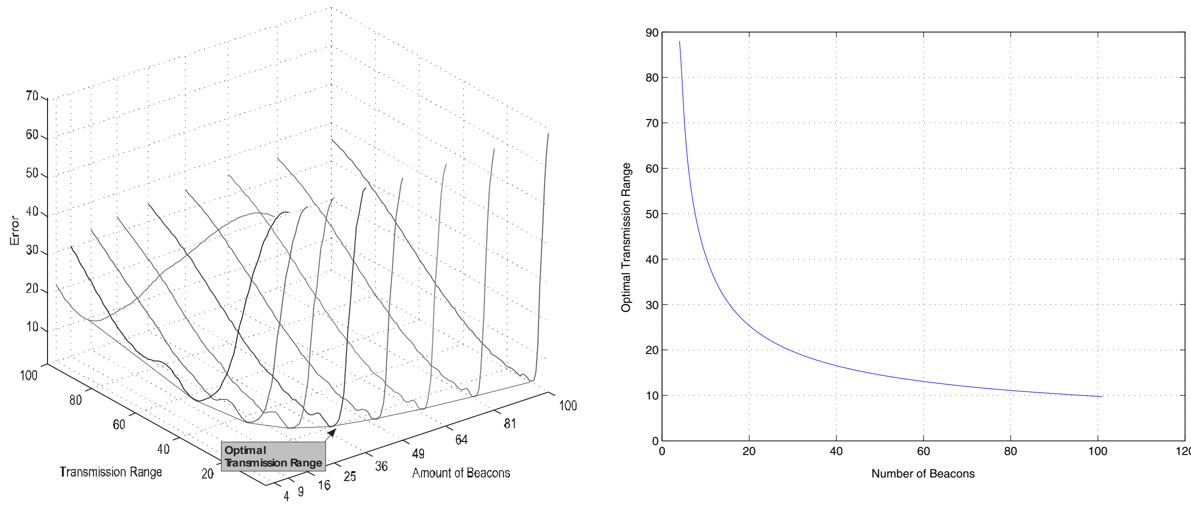

Figure 10. a) Dependency of positioning error on number of beacons and transmission range, b) Optimal transmission range over number of beacons, 101x101 nodes at array width=100

achieved at a granularity of 0.86. Fig. 10b demonstrates the optimal transmission range over number of beacons based on (20) using the experimentally found granularity G_{opt}=0.86.

The practical adaptability of our equations reveals in a simple realization and optimization of a sensor network with fixed beacon positions. By insertion of distance d in (20), the optimal transmission range of all beacons is determined. In an ideal environment, the correct transmission power E_{Trans} is calculated as represented in (21) using the optimal transmission range r_{opt}. This transmission power has to be adjusted directly in the transceiver module of each beacon.

$$E_{Trans} = E_{Bit} \cdot \left(\frac{4 \cdot \pi \cdot r_{opt}}{\lambda} \right)^2 \qquad (21)$$

E_{Trans} = *Energy required to transmit one bit depending on r*

E_{Bit} = *Base energy to transmit one bit*

λ = *Wave length*

r_{opt} = *Optimal transmission range*

In contrast to sending with a maximal transmission range, all beacons consume less energy and sensor nodes calculate their position with an optimal precision. Fundamentally at an optimal transmission range, each beacon transmits messages only to a prior determined area containing sensor nodes. Outside this area, all other sensor nodes do not receive messages from this beacon and safe energy. Hence, sensor network lifetime enlarges. In addition, a predefined transmission range reduces the necessity to configure the sensor network at runtime and; therefore, safes transmission energy, too.

Further on, our current considerations deal with the practical qualification of the algorithm. In our theoretical model, the transmission range of the beacons is defined as ideal circular. But in reality, the maximal transmission range fluctuates at different angles around the antenna. Therefore, unknowns near the boundary receive less beacon information and thus they calculate their own position more inaccurately. In the end, the average positioning error increases and some sensor nodes are not able to determine their own position.

7. Conclusion

We presented an analytic proof of a simple equation to improve the precision of Coarse Grained Localization with Centroid Determination in grid-aligned wireless sensor networks. The equation calculates the optimal transmission range of beacons in finite wireless sensor networks. Using this optimal transmission range, energy consumption and calculation overhead is reduced during position determination of simple nodes. Further on, the relative positioning error across the entire sensor network is reduced significantly.

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