Low Power Optimization of the "Coarse Grained Localization" Algorithm in Wireless Sensor Networks

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Outline

- Motivation
- Coarse Grained Algorithm
- Error and power minimization
  - Graphical approach
  - Power-Error product
  - Analytical derivation

Motivation

Position Determination

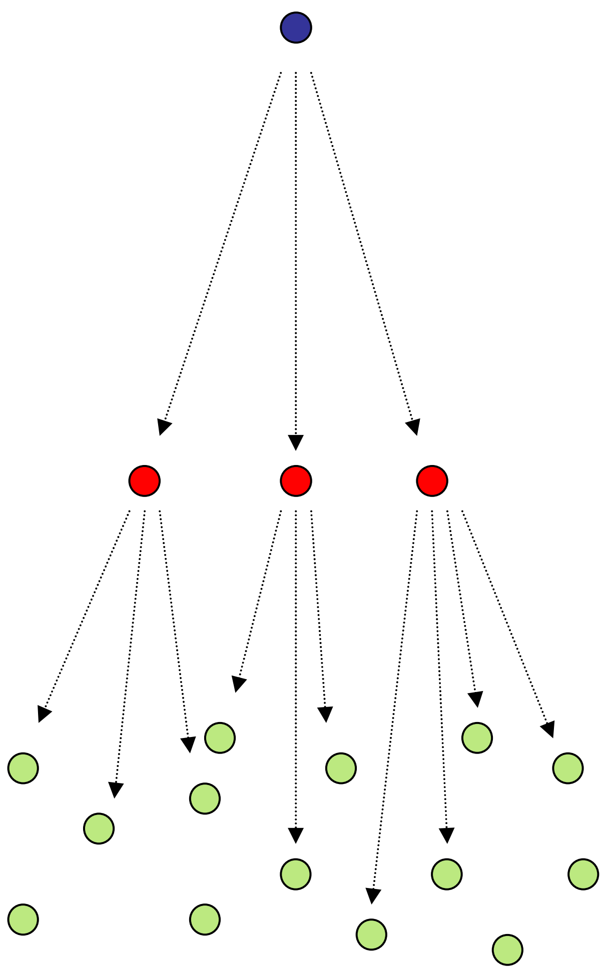
Why Positioning in Wireless Sensor Networks ?

- Sensor nodes are randomly deployed
- Measurements have to be assigned to their location
- Prerequisite for location-based services

Where is the leakage?

Hierarchy in Sensor Networks

- **Base station / Gateway:**
  - Immobile
  - Position is known
  - Resources uncritical
- **Beacons:**
  - Wireless (sensor) nodes
  - **Position is known**
  - Low resources
- **Simple sensor nodes:**
  - Wireless sensor nodes
  - **Position is not known**
  - Very low resources

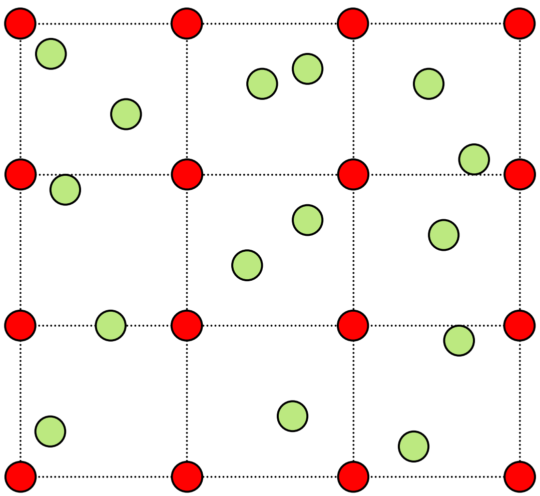


: Base station : Beacon : Simple Sensor Node

Our Model

Prerequisites:

- Square array of beacons
- Beacons are grid-aligned (Infrastructure distribution)
- Circular transmission range
- Randomly distributed sensor nodes



Our considerations:

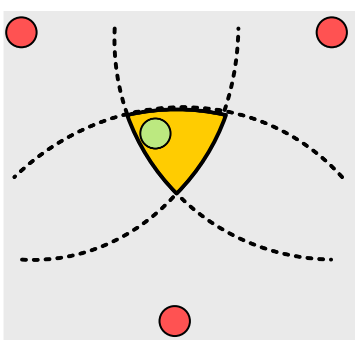
- Objectives
  - Small positioning error
  - Less energy consumption
  - Minimize number of beacons
- Simulations and analytical study

: Beacon : Sensor Node

Positioning Algorithms

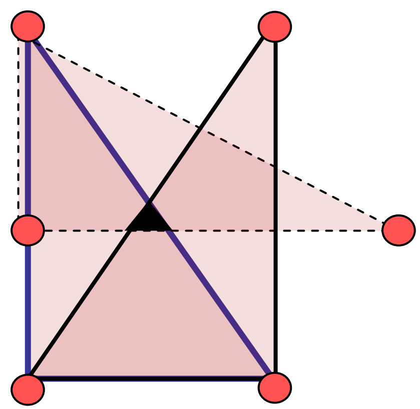
Position Determination

- Coarse-Grained, Δ>5%
- Fine-Grained, Δ<5%

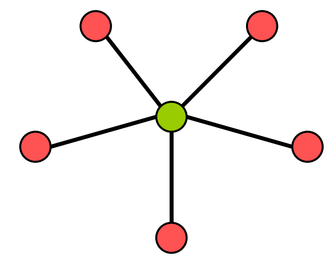


$$X_{Est} = \left(\frac{X_{i1} + \dots X_{ik}}{k}\right)$$

Centroid Determination (Coarse Grained)



Triangle overlapping (APIT)



$$\Delta_i(x_0, y_0, d_i) = d_i - \sqrt{(x_i - x_0)^2 - (y_i - y_0)^2}$$

$$\underline{b} = (\underline{X}^T \underline{X})^{-1} \underline{X}^T \underline{y}$$

Multilateration (Atomic, iterative, collaborative)

Δ : Positioning Error   ● : Beacon   ● : Sensor Node

Coarse Grained Algorithm

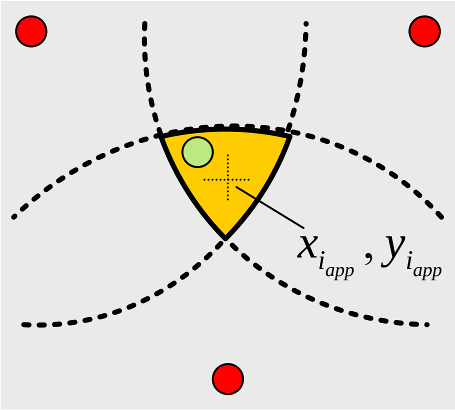
Algorithm: Coarse Grained (CGL)[1]

Position Determination:

- Sensor nodes receive messages containing position information of *n* different beacons
- Sensor nodes can be only located within an area achievable by all *n* beacons
- Position determination simple through centroid calculation

Advantages:

- No Received Signal Strength measurements
- No transmissions caused by sensor nodes (only reception)
- Simple calculation



$$x_{i_{app}}, y_{i_{app}} = \left(\frac{1}{n} \sum_{k=1}^{n} x_{B_k}, \frac{1}{n} \sum_{k=1}^{n} y_{B_k} \right)$$

: Beacon   : Sensor Node

: Target area of sensor node

: Transmission range of beacons

1) N. Bulusu, et. al.: „*GPS-less Low Cost Outdoor Localization For Very Small Devices*“

Positioning Error in CGL

Positioning Error:

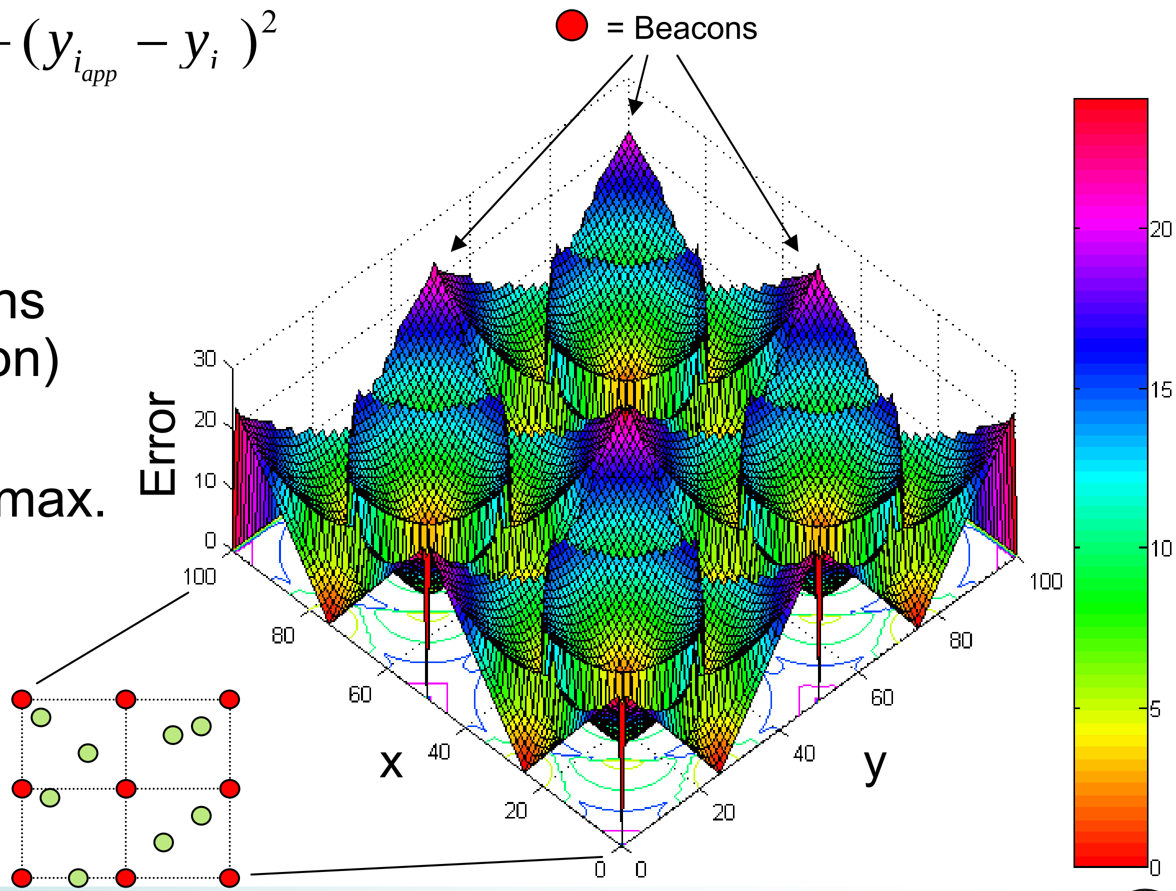
- Distance between calculated and real position

$$f_i(x,y) = \sqrt{(x_{i_{app}} - x_{i_a})^2 + (y_{i_{app}} - y_i)^2}$$

$x_{i_{app}}, y_{i_{app}}$ = calculated coordinates of node i

x_{i_a}, y_{i_a} = real coordinates of node i

f_i = Positioning error of node i



Error Behavior:

- Grid-aligned 3x3 beacons (Infrastructure distribution)
- Array size 100x100
- 101x101 sensor nodes max.
- Transmission range of beacons r = 50

Error and Power Optimization
- Graphical Approach -

Boundary Conditions

Averaged positioning error is max if transmission range *r* of beacons is

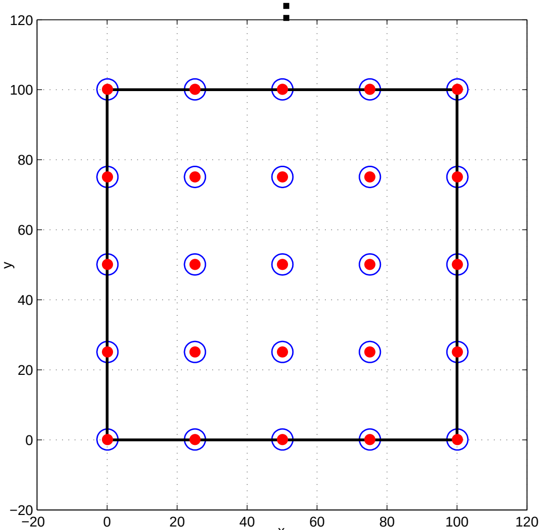
$r \to 0$ or $r >>$ array diagonal

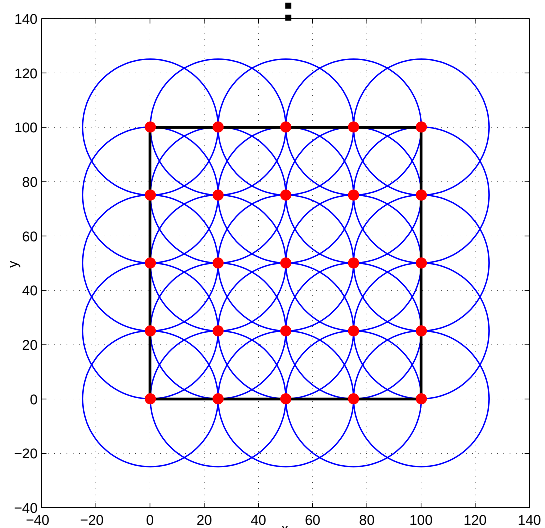
- Sensor nodes do not receive any beacon positions
- Position determination is not possible
- Number of unknowns is maximal

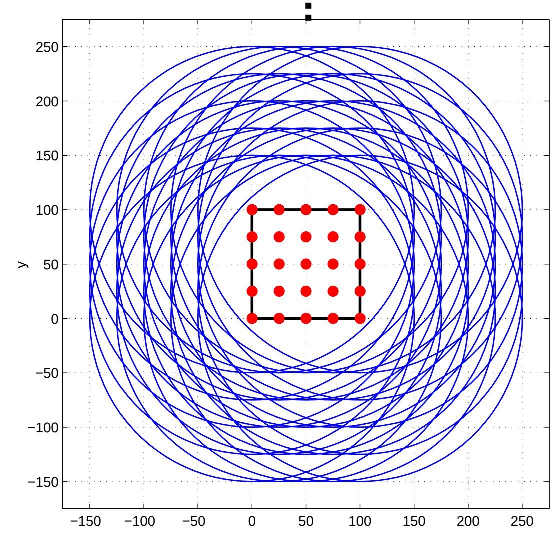
- All sensor nodes receive identical beacon positions
- All sensor nodes calculate identical own position (unknowns=0)

Looking for optimal transmission range *r* of beacons, where

- **Number of unknowns → 0**
- **Positioning error → min.**







: Transmission Range : Sensor Network : Beacon Array size: 100x100

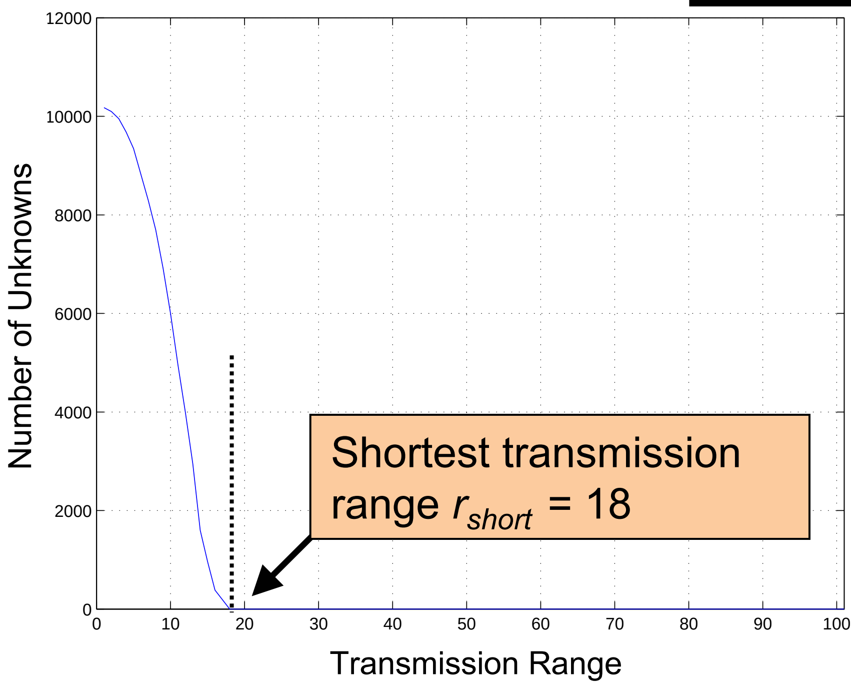
Graphical Solution

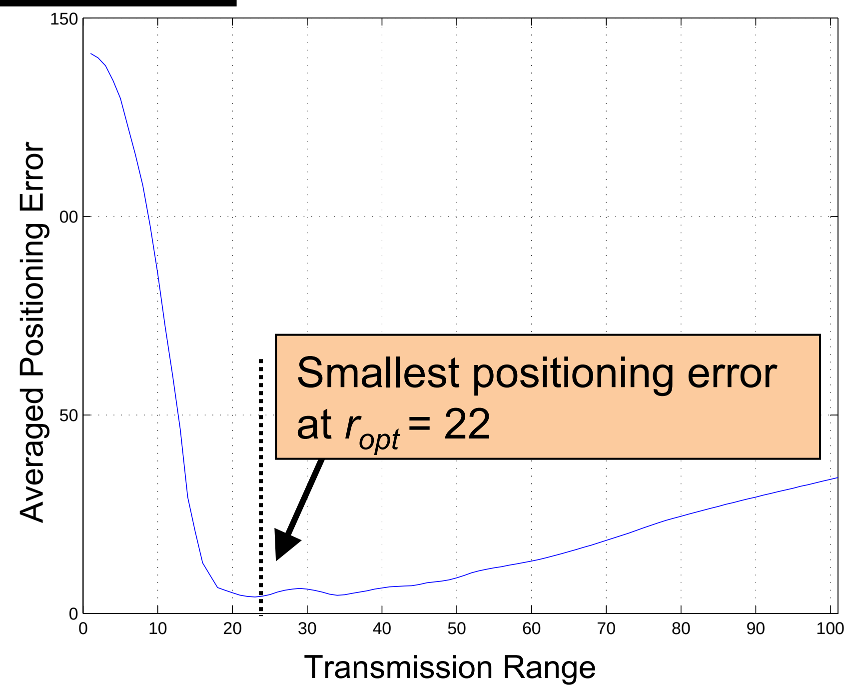
First Approach:

Determine shortest transmission range r_{small} where number of unknowns is zero.

But: Positioning error is not minimal!

$r_{short} < r_{opt}$





Power-Error Product (PEP)

Energy Considerations (ideal):

$$E_{Send} = E_{Init} + mE_{Dyn}$$

$$E_{Dyn} = E_{Bit} \cdot \left(\frac{4 \cdot \pi \cdot r}{\lambda} \right)^2$$

$$E_{Dyn}\Big|_{E_{Bit} \cdot \left(\frac{4\cdot\pi}{\lambda}\right)^2 = 1} = r^2$$

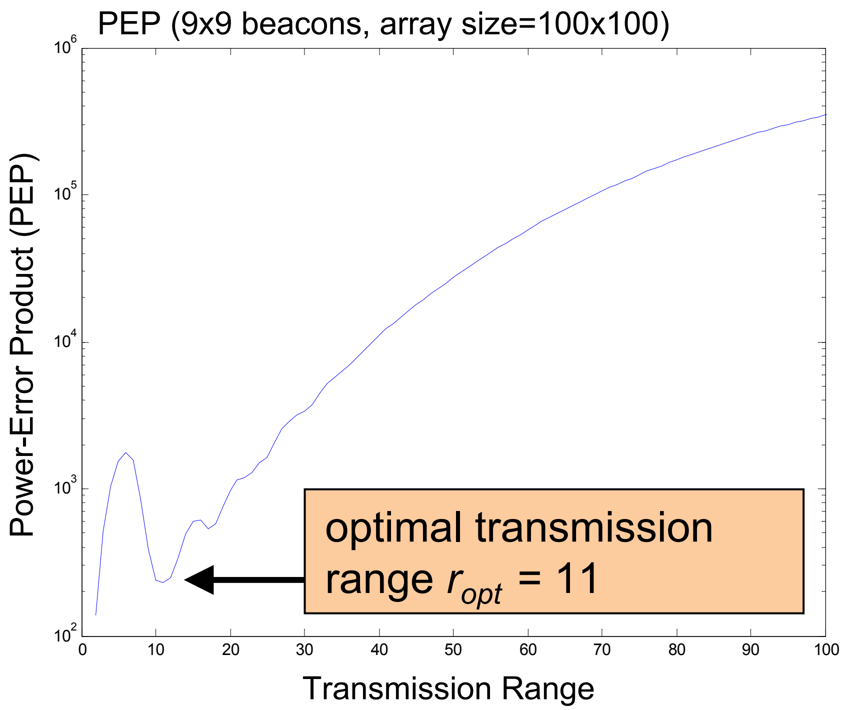
- m = Number of bits to transmit
- E_{Init} = Energy to initialize transmitter
- E_{Dyn} = Transmission energy of one bit
- λ = Wave length
- r = Transmission range of beacons
- PEP = Power-Error product
- L = Number of nodes in network
- r = Distance
- f_i = Positioning error of node
- $f_{l_{mean}}$ = Averaged positioning error

Power-Error Product:

$$PEP = E_{Dyn} \cdot f_{l_{mean}}$$

$$PEP = r^2 \cdot \frac{\sum_{i=1}^{l} f_i}{l}$$

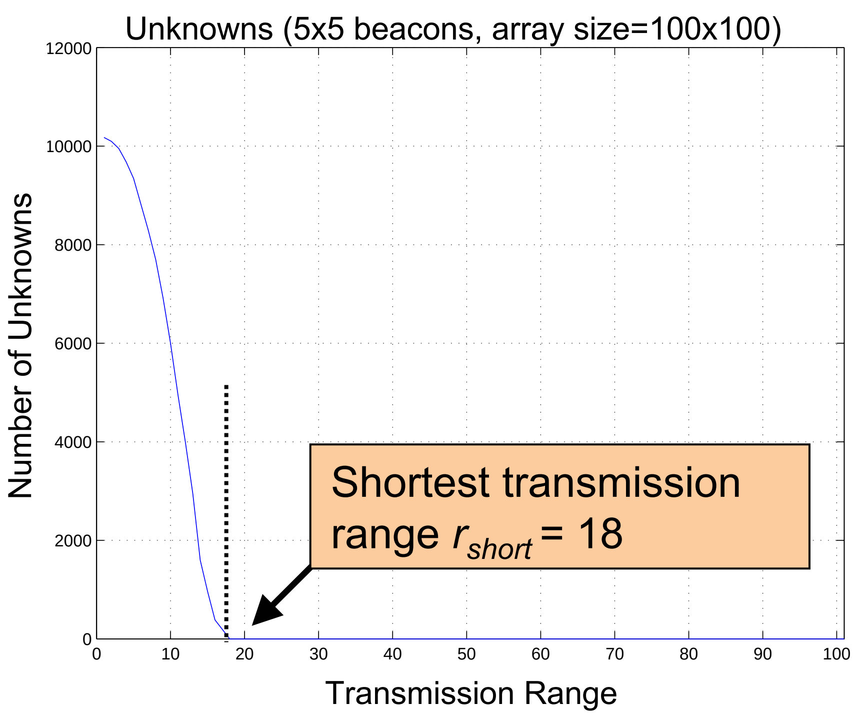
PEP is a performance indicator for the optimization problem of both transmission range and positioning error.

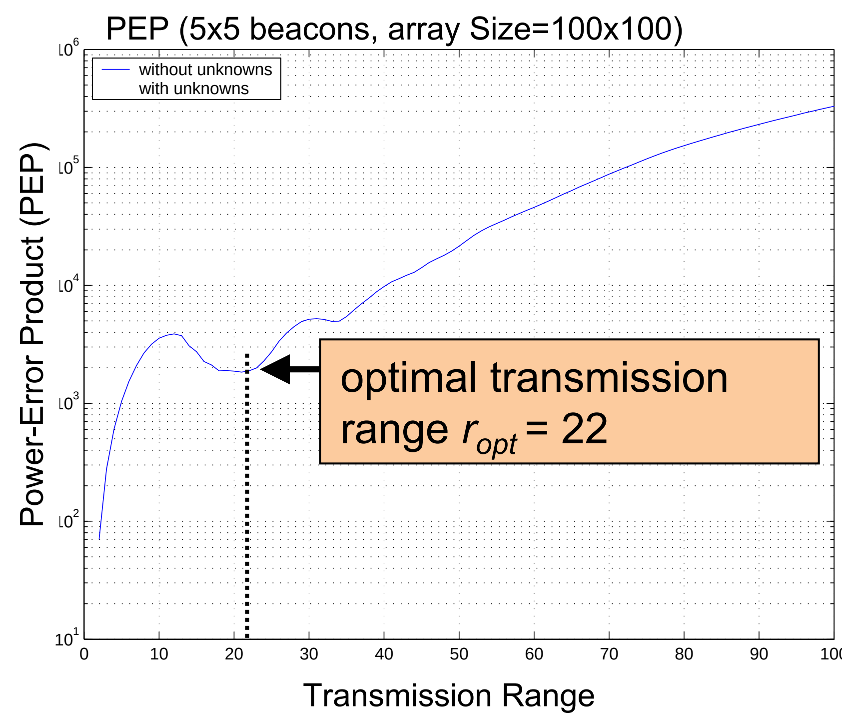


Graphical Solution II

Determine optimal transmission range:

- Identifying r_{short} (unknowns=0)
- Looking for the 1st PEP-Minimum for $r_{short} \leq r_{opt}$ (Example: 18 ≤ 22)





Additional Dependencies

Knowledge:

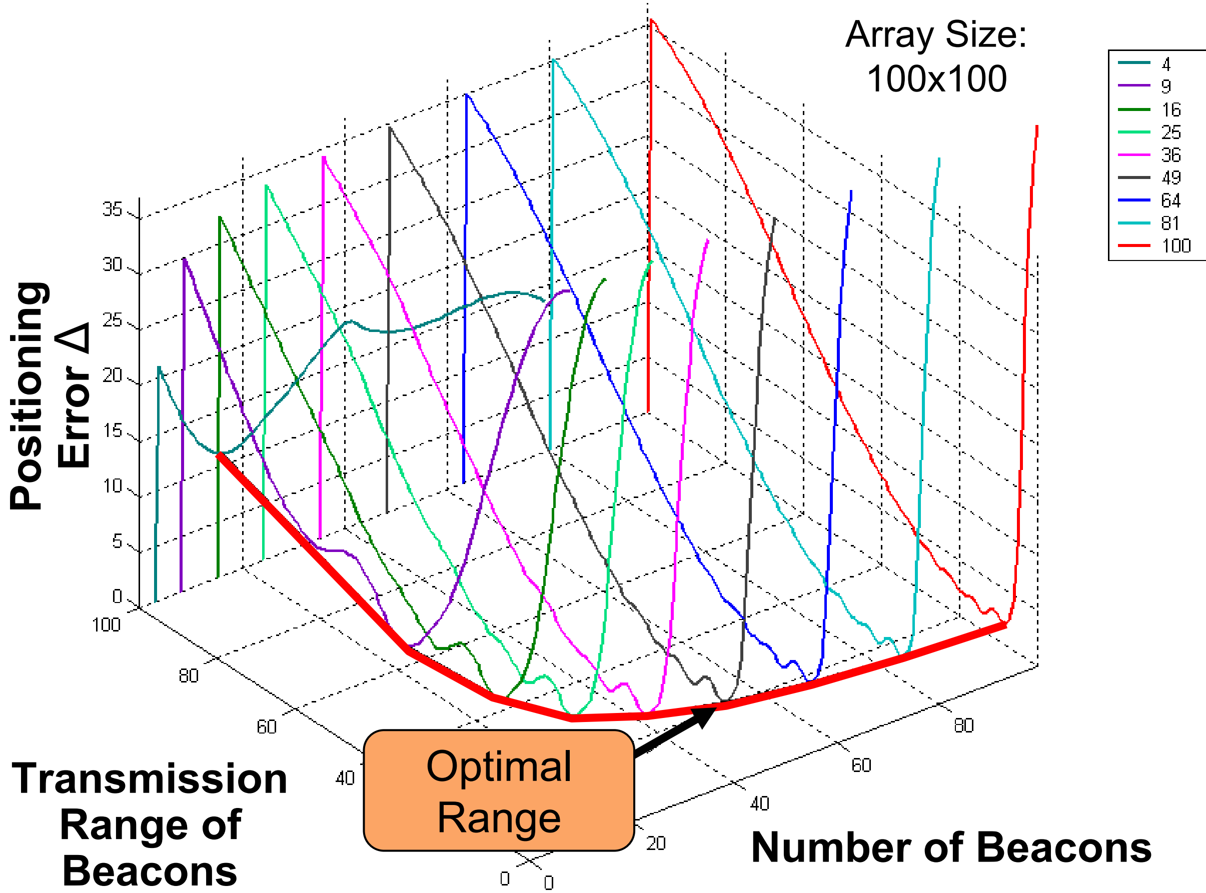
Optimal transmission range *r* varies with number of beacons *n*.

Relationship

r_{opt} = f(Δ, #Beacons)

Application

Energy Minimization



Error and Power Optimization
- Analytical Approach -

Distance of Beacons

Definitions:

d = distance between two beacons
w = array size
n = number of beacons

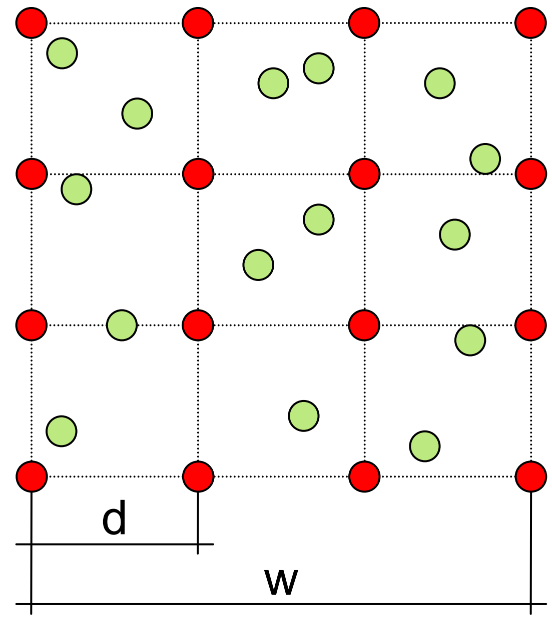
Equation 1:

$$d = \frac{w}{\sqrt{n} - 1}$$

Example:

$$d = \frac{150}{\sqrt{16} - 1} = \frac{150}{3} = 50$$

Sensor Network (w=150, n=16)

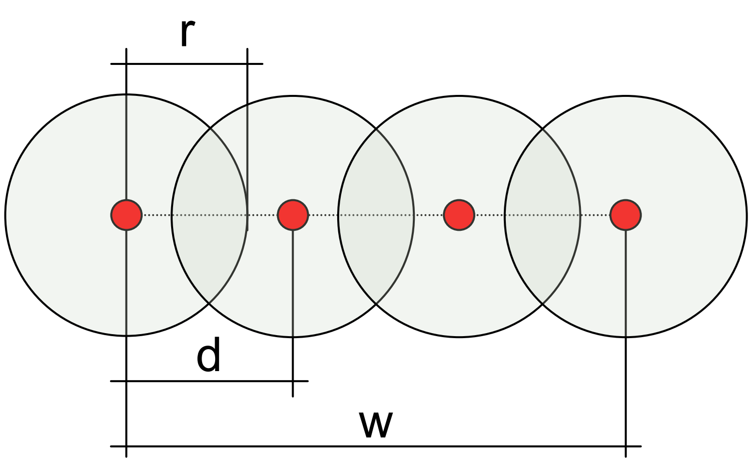


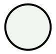
● : Beacon ● : Sensor Node

Granularity

Granularity:

- Ratio of transmission range of beacons *r* to distance *d*
- Independent of
  - array size *w*
  - number of beacons *n*

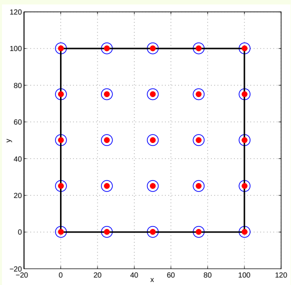


d = distance between two beacons
w = array size
r = transmission range
G = granularity
 = transmission area
 = Beacon

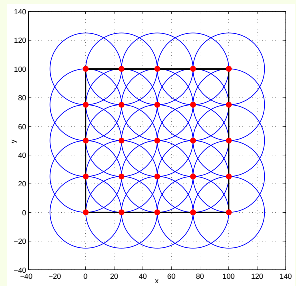
Equation 2: $G = \frac{r}{d}$

Optimal Granularity

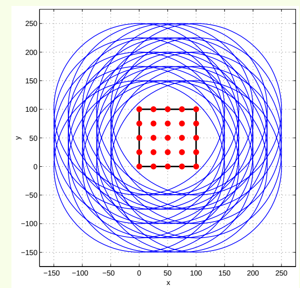
r→0



r=optimal



r>>diagonal



Δ = high
G = low

Δ = low
G = optimal

Δ = high
G = high

$$G_{opt} = \frac{r_{opt}}{d}$$ → **G_{opt} exists!**

Optimal Granularity:

- Independent of array size *w* and number of beacons *n*
- G_{opt} = constant
- Can be used to determine r_{opt}

Δ : Positioning Error   ● : Beacon   ○ : Transmission Range of Beacon

Optimal Transmission Range

Note:

$$d = \frac{w}{\sqrt{n} - 1}$$

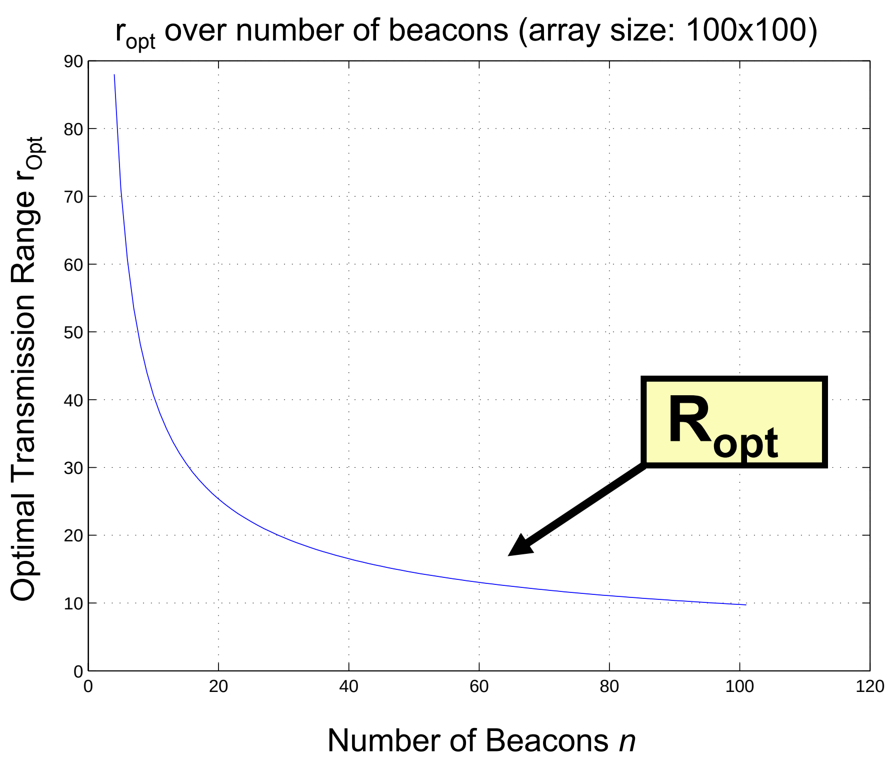
d = distance between two beacons
w = array size
n = number of beacons
r = transmission range
G = granularity

Derivation of r_{opt}:

$$G_{opt} = \frac{r_{opt}}{d}$$

$$r_{opt} = G_{opt}\, d$$

$$r_{opt}(w, n) = \frac{G_{opt} \cdot w}{\sqrt{n} - 1}$$

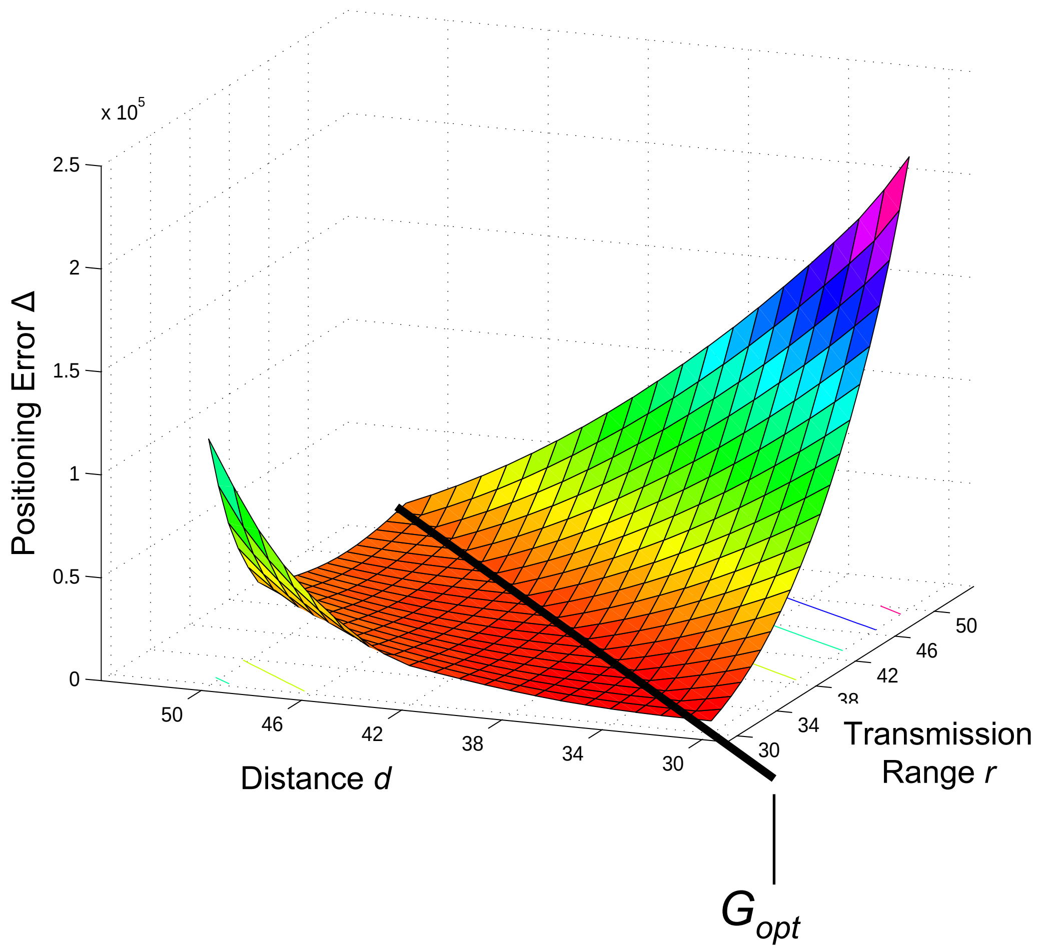


Determination of G_{Opt}

How do we determine G_{opt} ?

- Calculating positioning error Δ_i by varying *r* and *d*
- Determining r_{opt} at all distances d_i with smallest positioning error

$$G_{opt_i} = \frac{r_{opt_i}}{d_i}$$



Results:

- G_{opt} is constant !
- $G_{opt} \approx 0.88$

Conclusion

Results

- Positioning error depends on number of beacons and transmission range
- Graphical and analytical solutions to determine optimal transmission range
- Constant Granularity $G_{opt} = 0.88$
- Optimality criterion: Power-Error Product

To Do

- Analytical proof of presented equations to determine r_{opt}
- Mathematical analysis of stochastically distributed beacons
- Extension to non-circular transmission ranges

Thank you