

Exercise 10: Convergence

Summer Term 2024

Review: Questions:

1. What does the term *convergence* mean?
2. What does the term *progress* mean?
3. How did we define the term “rate of progress φ ”?
4. How is the steepest-descent method defined?

To Do: For the steepest-descent method, you should calculate the rate of progress φ . Furthermore, you should determine the necessary conditions for convergence.

Tasks:

Steepest Descent:

Consider the one-dimensional quadratic fitness function $f(x) = 0.5x^2$.

1. Please calculate the gradient $\nabla f(x) = f'(x)$.
2. Please calculate the rate of progress $\varphi = \dots$
3. Under which condition, i.e., values for η , does the steepest-descent method converge?
4. What is the relationship between convergence and the quotient $\zeta = \frac{|x^{t+1}|}{|x^t|}$?
5. Do you need to adapt the step size σ ?
6. Please, complete the table and figures presented below for the following values: $\eta = 0.5, \eta = 1.5$, and $\eta = 1.0$. You may always start at $x^{t=0} = 32$.

What happens for $\eta < 0, \eta = 0, \eta = 2.0$, and $\eta > 2.0$?

Please, indicate the different regimes in the last figure.

Optional: if you want to do so, generalize your result to the n -dimensional case.

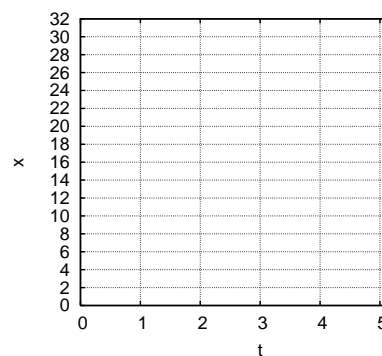
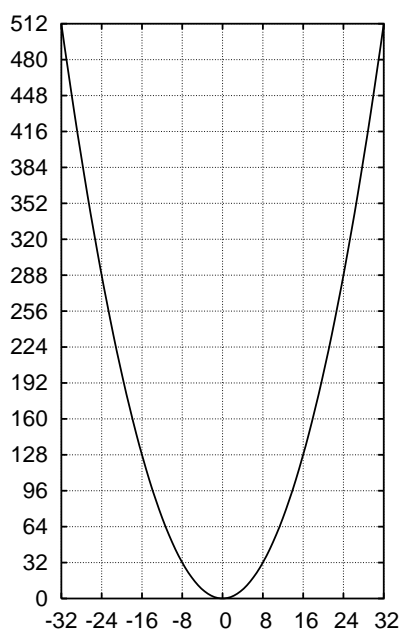
Now, do the same steps for the function $f(x) = 0.25x^4$.

1. Please calculate the gradient $\text{grad}f(x) = \nabla f(x) = f'(x)$.
2. Please calculate the rate of progress $\varphi = \dots$
3. Under which condition does the steepest-descent method converge?

4. Do you need to adapt the step size σ ?

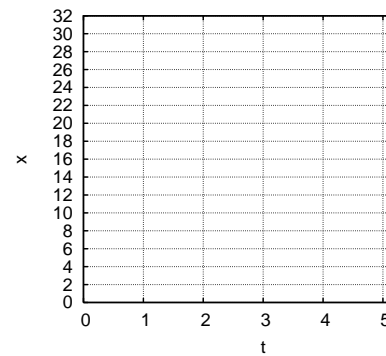
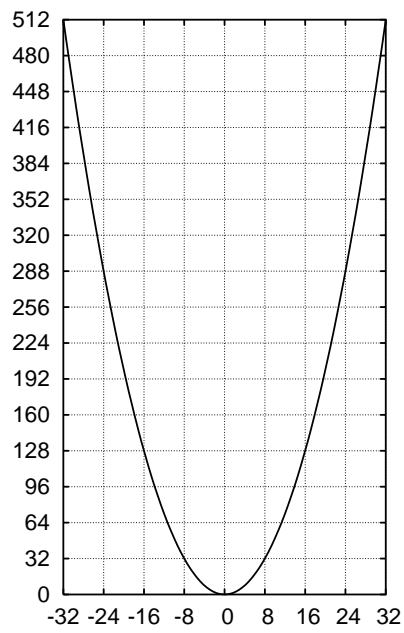
$$f(x) = x^2, x^{t=0} = 32, \eta = 0.5$$

	1	2	3	4	5	5	7
x^t							
$f(x^t)$							
grad $f(x^t)$							



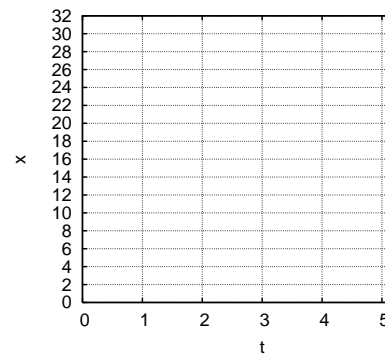
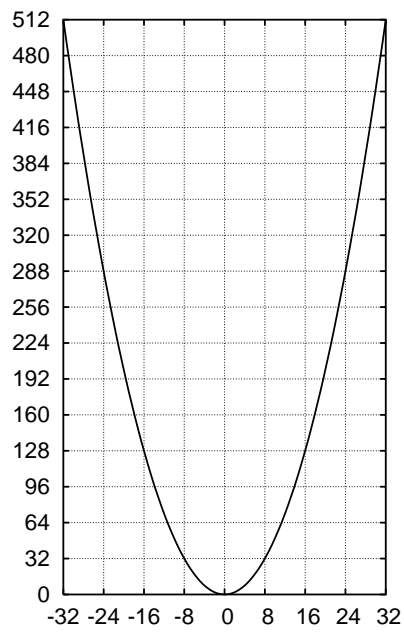
$$f(x) = x^2, x^{t=0} = 32, \eta = 1.5$$

	1	2	3	4	5	5	7
x^t							
$f(x^t)$							
grad $f(x^t)$							

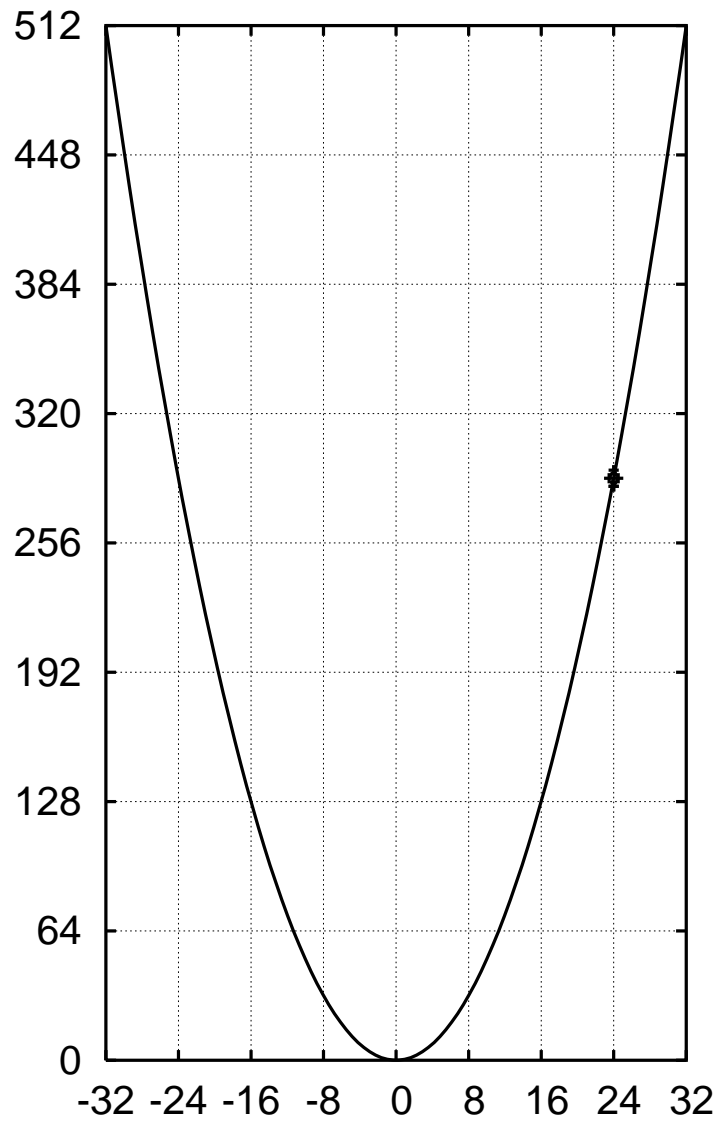


$$f(x) = x^2, x^{t=0} = 32, \eta = 1.0$$

	1	2	3	4	5	5	7
x^t							
$f(x^t)$							
grad $f(x^t)$							



The different regimes at $x^{t=0} = 32$



Have fun, Theo and Ralf.